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The strangeness content of the nucleon from effective field theory and phenomenology

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ABSTRACT

We revisit the classical relation between the strangeness content of the nucleon, the pion–nucleon sigma term and the $SU(3)_F$ breaking of the baryon masses in the context of Lorentz covariant chiral perturbation theory with explicit decuplet-baryon resonance fields. We find that a value of the pion–nucleon sigma term of ∼60 MeV is not necessarily at odds with a small strangeness content of the nucleon, in line with the fulfillment of the OZI rule. Moreover, this value is indeed favored by our next-to-leading order calculation. We compare our results with earlier ones and discuss the convergence of the chiral series as well as the uncertainties of chiral approaches to the determination of the sigma terms.

1. Introduction

We dedicate this study to the interplay between the nucleon sigma terms, $\sigma_{\pi N}$ and σ_s, which are defined as

$$\sigma_{\pi N} = \frac{1}{2M_N}\langle N|\hat{m}(\bar{u}u + \bar{d}d)|N\rangle,$$
$$\sigma_s = \frac{1}{2M_N}\langle N|m_s\bar{s}s|N\rangle. \tag{1}$$

Here, the up, down and strange quarks masses are indicated by m_u, m_d and m_s, respectively, and $\hat{m} = (m_u + m_d)/2$. In the following, we restrict ourselves to the isospin limit, $m_u = m_d = \hat{m}$, with the nucleon states having the Lorentz invariant normalization $\langle N(\mathbf{p}', s')|N(\mathbf{p}, s)\rangle = 2E_N(2\pi)^3\delta(\mathbf{p}' - \mathbf{p})$, where $E_N = \sqrt{M_N^2 + \mathbf{p}^2}$, M_N is the nucleon mass and s and s' are the spin indices.

Both $\sigma_{\pi N}$ and σ_s are interesting observables and their non-vanishing values would clearly indicate that quark masses are not zero and give contribution to the nucleon mass. More precisely, the values of these two sigma terms embody the internal scalar structure of the proton and neutron. If they are small, most of the nucleon mass stems from the confinement of the lightest quarks in typical distances around 1 fm. Another property related to the nucleon scalar structure is the strangeness content of the nucleon, y, which is defined as

$$y = \frac{2\langle N|\bar{s}s|N\rangle}{\langle N|\bar{u}u + \bar{d}d|N\rangle} = \frac{2\hat{m}}{m_s}\frac{\sigma_s}{\sigma_{\pi N}}. \tag{2}$$

Notice that if the OZI rule (large N_C prediction) were exact then $y = 0$. Besides their role in understanding the mass of the ordinary matter, $\sigma_{\pi N}$ and σ_s are also necessary with respect to theoretical speculations on the origin of dark matter particles based on supersymmetry. An accurate determination of the sigma terms is needed to constrain the parameter space of the underlying supersymmetric models from the experimental bounds in direct searches of weakly interacting dark matter particles [1].

The determination of $\sigma_{\pi N}$ is feasible from πN scattering data due to the low-energy theorem of current algebra [2] that relates the value of the isospin even πN scattering amplitude at the Cheng–Dashen point with the nucleon scalar form factor [3–5]. However, the situation is much more obscure for the strangeness scalar form factor of the nucleon, and then for the phenomenological determination of σ_s as well as of y. Historically [6], the path to escape this end point is based on combining the definitions of Eqs. (1) and (2) as

$$\sigma_{\pi N} = \frac{\sigma_0}{1 - y}, \tag{3}$$

where σ_0 is the nucleon expectation value of the purely octet operator $\bar{u}u + \bar{d}d - 2\bar{s}s$,

$$\sigma_0 = \frac{\hat{m}}{2M_N}\langle N|\bar{u}u + \bar{d}d - 2\bar{s}s|N\rangle. \tag{4}$$

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The point to notice is that the latter operator is the only one in the QCD Lagrangian responsible for the hadronic mass splitting within an $SU(3)$ multiplet. From the experimental values of the lightest baryon octet masses, M_Ξ, M_Σ and M_N, we can then calculate approximately σ_0 by making use of $SU(3)$ flavor symmetry, with the result [6]

$$\sigma_0 = \frac{\hat{m}}{m_s - \hat{m}}(M_\Xi + M_\Sigma - 2M_N) \simeq 27\ \text{MeV}, \tag{5}$$

where we have used $m_s/\hat{m} = 26(4)$ [7].

Additionally, with this value for σ_0 and by assuming the OZI rule to hold, so that $y = 0$, one obtains from Eq. (3) the naive estimation $\sigma_{\pi N} \simeq 30$ MeV, that is much smaller than its phenomenological determinations from πN scattering data. For instance, Gasser et al. [5] obtained the canonical result $\sigma_{\pi N} \simeq 45$ MeV [5] in terms of a dispersive analysis of the pre-90s πN elastic scattering data.[1] A partial-wave analysis including the more modern πN database carried out by the George Washington University group [8], resulted in larger values of the pion–nucleon sigma term, $\sigma_{\pi N} = 64(8)$ MeV [9]. Besides that, a study of πN elastic scattering in Lorentz covariant baryon chiral perturbation theory (BχPT) [10] agrees with the dispersive results, which depend on the data set employed [11]. Additionally, it also reveals that modern partial-wave analyses are, in general, more consistent with different scattering phenomenology than the older ones and lead to a relatively large value of the sigma-term, cf. $\sigma_{\pi N} = 59(7)$ MeV [11]. The actual value of $\sigma_{\pi N}$ has important consequences on the strangeness content of the proton since, according to Eq. (3) and the result for σ_0 in Eq. (5), all these values for $\sigma_{\pi N}$ extracted from πN scattering data would imply a very large result for y.

Now, at this point it is important to emphasize that Eq. (5) is an estimate obtained at leading order in an $SU(3)_F$-breaking expansion and the calculation of σ_0 from this equation could be affected by large higher order contributions. The next-to-leading order (NLO) chiral corrections were first calculated by Gasser in Ref. [12]. There he obtained $\sigma_0 = 35(5)$ MeV by employing a chiral model for the meson cloud around the baryon which only considered contributions from the virtual octet baryons. Within the more evolved theoretical framework of BχPT Ref. [13] performed a calculation of the baryon masses and σ_0 in the heavy-baryon (HB) [14] expansion up to next-to-next-to-leading order (NNLO). In this work, the contributions of the decuplet-baryon resonances were not implemented explicitly but through resonance-saturation hypothesis they contributed to several of the many low-energy-constants (LECs) appearing at this order. All in all, they reported the value $\sigma_0 = 36(7)$ MeV, which was almost identical to the NLO result obtained by Gasser 15 years earlier. Later, Ref. [15] also included the decuplet-baryon resonances within HBχPT using a cut-off regularization scheme and still obtained basically the same result for σ_0. One should also notice that $\sigma_{\pi N} = 45$ MeV was taken as input in the analyses of Refs. [13,15], which had a strong influence in the results of Ref. [15].

By employing $\sigma_0 \simeq 35$ MeV from the calculations of Refs. [12,13,15] in Eq. (3) one obtains that $y \simeq 0.2$ and 0.4 for $\sigma_{\pi N} \simeq 45$ MeV and $\simeq 60$ MeV, respectively. In particular, the latter value would imply a strangeness contribution to the mass of the nucleon of $\sim$300 MeV. Although not impossible, such a scenario with a strong breaking of the OZI rule is theoretically implausible, moreover after the experimental evidence pointing to a negligible strangeness contribution in other properties of the nucleon such as its electromagnetic structure [16] and spin [17]. Thus, if one gives credit to these results and translate them into a small value of y, then the present widely accepted value for σ_0 around 35 MeV clearly discredits the relatively large values for $\sigma_{\pi N}$ favored by the most recent analysis of the πN scattering data, cf. $\sigma_{\pi N} = 64(8)$ MeV [9] and $\sigma_{\pi N} = 59(7)$ MeV [11].

It is our aim in this work to emphasize that the situation concerning σ_0 is not settled yet, so that the previous conclusion does not necessarily hold. On one hand, the result of Gasser [12] is based on a model calculation of the meson cloud around the nucleon, whereas Refs. [13,15] might be afflicted by the poor convergence of the chiral series typically shown by HB in the $SU(3)_F$ theory [18,19].

A suitable approach that also includes explicitly the contributions from the decuplet-baryon resonances is a Lorentz covariant formulation of BχPT with a consistent power-counting via the extended-on-mass-shell renormalization (EOMS) scheme [20]. The relativistic corrections that results in this approach, in a way preserving the exact analytical properties of the Green functions, have been shown to tame the poorly convergent series of the HB expansion in baryonic observables as important as the magnetic moments [18,21] or masses [19,22]. Moreover, once a prescription is taken to treat the problem of the interacting Rarita–Schwinger fields [23], this scheme is straightforwardly applicable to include the contributions of the decuplet-baryon resonances [21]. In this work we calculate σ_0 up to NLO using Lorentz covariant BχPT renormalized in the EOMS prescription and including explicitly the effects of the decuplet. We compare the results with those obtained in the HB expansion and estimate systematic higher-order effects through a partial calculation of NNLO pieces. All together, we find the remarkable result that the value of σ_0 becomes larger so that the modern experimental determinations of $\sigma_{\pi N} \sim 60$ MeV are then consistent with a small strangeness content of the nucleon, or with a small OZI rule violation. A first indication that the decuplet contributions could help to solve the strangeness puzzle concerning a relatively large $\sigma_{\pi N}$ was given by the HB calculation in Ref. [24]. Indirectly, this was also the case in Ref. [25] where very large and negative values of σ_s were obtained when demanding $\sigma_{\pi N} = 45$ MeV, indicating a larger σ_0.

2. Calculation

The expressions for the sigma terms can be obtained either from the explicit calculation of the scalar form factor of the nucleon at $q^2 = 0$ or applying the Hellmann–Feynman theorem to the chiral expansion of its mass,

$$\begin{aligned}\sigma_{\pi N} &= \hat{m}\frac{\partial M_N}{\partial \hat{m}} = \frac{m_\pi^2}{2}\left(\frac{1}{m_\pi}\frac{\partial}{\partial m_\pi} + \frac{1}{2m_K}\frac{\partial}{\partial m_K} + \frac{1}{3m_\eta}\frac{\partial}{\partial m_\eta}\right)M_N \\ &\quad + \mathcal{O}(p^4), \\ \sigma_s &= m_s\frac{\partial M_N}{\partial m_s} = \left(m_K^2 - \frac{m_\pi^2}{2}\right)\left(\frac{1}{2m_K}\frac{\partial}{\partial m_K} + \frac{2}{3m_\eta}\frac{\partial}{\partial m_\eta}\right)M_N \\ &\quad + \mathcal{O}(p^4).\end{aligned} \tag{6}$$

We follow the latter strategy since the explicit expressions for the baryon masses in the different schemes treated in this Letter can be directly obtained using the Appendix of Ref. [19]. Thus, the chiral expansion of the sigma terms up to NLO from Eq. (6) is written as,

$$\begin{aligned}\sigma_{\pi N} &= -4(2b_0 + b_D + b_F)\frac{m_\pi^2}{2} \\ &\quad + \frac{1}{(4\pi F_\phi)^2}\sum_{\phi=\pi,K,\eta}\left(\xi_{N,\phi}^{(B)}\Sigma_\pi^{(B)}(m_\phi) + \xi_{N,\phi}^{(T)}\Sigma_\pi^{(T)}(m_\phi)\right) \\ &\quad + \mathcal{O}(p^4),\end{aligned}$$

[1] For a detailed exposition of the dispersive methods for obtaining $\sigma_{\pi N}$ from the analytic continuation of the πN scattering amplitude to the Cheng–Dashen point see Refs. [3–5].