



# Regular black hole metrics and the weak energy condition



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## ABSTRACT

In this work we construct a family of spherically symmetric, static, charged regular black hole metrics in the context of Einstein–nonlinear electrodynamics theory. The construction of the charged regular black hole metrics is based on three requirements: (a) the weak energy condition should be satisfied, (b) the energy–momentum tensor should have the symmetry  $T_0^0 = T_1^1$ , and (c) these metrics have to asymptotically behave as the Reissner–Nordström black hole metric. In addition, these charged regular black hole metrics depend on two parameters which for specific values yield regular black hole metrics that already exist in the literature. Furthermore, by relaxing the third requirement, we construct more general regular black hole metrics which do not behave asymptotically as a Reissner–Nordström black hole metric.

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## 1. Introduction

Charged regular black hole solutions exist in the framework of Einstein–nonlinear electrodynamics theory and are obtained as solutions of Einstein equations that are characterized by the fact that the metric as well as the curvature invariants  $R$ ,  $R_{\mu\nu}R^{\mu\nu}$ ,  $R_{\kappa\lambda\mu\nu}R^{\kappa\lambda\mu\nu}$  do not present singularities anywhere.<sup>1</sup>

The Bardeen black hole is the first of a series of regular black hole solutions obtained [2]. If we write the most general form of a static line element with spherical symmetry

$$ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2), \quad (1)$$

where the metric function can be written as

$$f(r) = 1 - \frac{2m(r)}{r}, \quad (2)$$

then, we can write the mass function of the Bardeen black hole as [2]<sup>2</sup>

$$m(r) = \frac{Mr^3}{(r^2 + g^2)^{3/2}}. \quad (3)$$

Such a metric has event horizons located at  $r_{\pm}$  if  $g^2 \leq (16/27)M^2$ , where  $g$  can be interpreted as the monopole charge of a self-gravitating magnetic field described by nonlinear electrodynamics [3]. Furthermore, if  $r \rightarrow \infty$ , we get that the metric behaves as

$$f(r) \rightarrow 1 - \frac{2M}{r} + \frac{3Mg^2}{r^3}. \quad (4)$$

Similarly, when  $r \rightarrow 0$  the Bardeen metric function behaves as the de Sitter black hole, that is as

$$f(r) \rightarrow 1 - \frac{2M}{g^3}r^2. \quad (5)$$

Later on, other solutions [4–8] were obtained, using the F–P dual formalism, by considering the action of general relativity coupled to nonlinear electrodynamics, namely

$$S = \int d^4x \sqrt{-g} \left( \frac{1}{16\pi} R - \frac{1}{4\pi} L(F) \right). \quad (6)$$

Here, the Lagrangian  $L(F)$  is a nonlinear function of the Lorentz invariant  $F = \frac{1}{4}F^{\mu\nu}F_{\mu\nu}$  which, for weak fields, describes the Maxwell theory, and the corresponding regular black hole solution asymptotically behaves as the Reissner–Nordström black hole.

The regular black hole may also be characterized by energy conditions [9,10] that the corresponding energy–momentum tensor should satisfy. In the context of regular black holes, three energy conditions have been utilized by several authors. In particular:

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<sup>1</sup> For a recent review see [1].

<sup>2</sup> Henceforth, we use geometrized units, i.e.,  $G = c = 1$ .

- The strong energy condition (SEC) states that  $T_{\mu\nu}t^\mu t^\nu \geq \frac{1}{2}T^\mu{}_\mu t^\nu t_\nu$  for all timelike vector  $t^\mu$ . This condition implies that gravitational force is attractive.
- The dominant energy condition (DEC) states that  $T_{\mu\nu}t^\mu t^\nu \geq 0$  and that  $T^{\mu\nu}t_\mu$  must be non-spacelike for all timelike vector  $t^\mu$  or, equivalently,  $T^{00} \geq |T^{ij}|$  for each  $i, j = 1, 2, 3$ . This means that the local energy density measured by a given observer must be nonnegative, and that the speed of the energy density flow associated with this observer cannot exceed the speed of light.
- The weak energy condition (WEC) states that the energy density of matter measured by an observer, whose 4-velocity is  $t^\mu$ , satisfies  $T_{\mu\nu}t^\mu t^\nu \geq 0$  for all timelike vector  $t^\mu$ , that is, the local energy density cannot be negative for all observers. The DEC implies the WEC.

When a black hole is regular, the SEC is necessarily violated somewhere inside the horizon [11]. However, a regular black hole could satisfy the WEC or the DEC everywhere [8].

Considering the line element (1) and the metric function (2), we can write the components of the respective energy-momentum tensor as

$$T^0_0 = T^1_1 = \frac{2}{8\pi r^2} \frac{dm(r)}{dr}, \quad T^2_2 = T^3_3 = \frac{1}{8\pi r} \frac{d^2m(r)}{dr^2}, \quad (7)$$

and hence the WEC can be expressed equivalently in terms of the mass function by the following inequalities

$$\frac{1}{r^2} \frac{dm(r)}{dr} \geq 0, \quad (8)$$

and

$$\frac{2}{r} \frac{dm(r)}{dr} \geq \frac{d^2m(r)}{dr^2}. \quad (9)$$

From these relations, it is obvious that the Bardeen black hole satisfies the WEC everywhere [2]. The same applies to regular black hole solutions reported in Refs. [4,7,8,12–15]. However, there are other regular black hole solutions reported in Refs. [5–7,16,17], which do not satisfy the WEC.

The regular black hole solutions that satisfy the WEC and their energy-momentum tensor has the symmetry  $T^0_0 = T^1_1$ , necessarily have de Sitter behavior at  $r \rightarrow 0$  as was shown in Ref. [13], and illustrated here in Eq. (5) for the Bardeen black hole solution. As we will see later, this leaves an extra condition which allows the WEC to be better exploited compared to the DEC when one is building solutions. However, it should be noted that a de Sitter behavior at the center of a regular black hole is not sufficient by itself to ensure that the solution satisfies the WEC. In addition, it has been shown that for a regular black hole in nonlinear electrodynamics which satisfies the WEC, the non-existence theorems [16] can be circumvented by removing the condition of the Maxwell weak field limit imposed at the center of the black hole. In this way, regular black hole solutions with electric charge do exist [8].

Furthermore, there are other features that characterize regular black holes which are due to the nonlinearities of the field equations. For example, the thermodynamic quantities of the regular black holes do not satisfy the Smarr formula [18], the identity of Bose–Dadhich [19] which refers to the relation between the Brown–York energy and the Komar charge, is not satisfied by regular black holes [20].

In all the above-mentioned solutions, one asymptotically recovers the Schwarzschild black hole metric, and if the condition  $r^2m'(r) \neq 0$  as  $r \rightarrow \infty$  is satisfied, then one recovers the Reissner–Nordström black hole metric.

In this Letter, in the context of Einstein–nonlinear electrodynamics theory, we will construct a family of spherically symmetric, static, charged regular black hole metrics without utilizing the aforesaid methods but by imposing three conditions: (a) the weak energy condition should be satisfied, (b) the energy–momentum tensor should have the symmetry  $T^0_0 = T^1_1$ , and (c) these metrics have to asymptotically behave as the Reissner–Nordström black hole metric. In addition, by relaxing the third condition, i.e., condition (c), we construct more general regular black hole metrics which do not behave asymptotically as a Reissner–Nordström black hole metric. The Letter is organized as follows. In Section 2, we present the equations that we will use in the construction of our metrics. In Section 3, we obtain a general metric for charged regular black holes that satisfies the WEC and asymptotically behaves as the Reissner–Nordström black hole metric. In addition, we discuss a specific case of the aforesaid general metric function. In Section 4, we extend the analysis to obtain metric functions that do not necessarily asymptotically behave as the Reissner–Nordström solution. Finally, in Section 5, we briefly summarize our results.

## 2. WEC equations

Up to now, regular black holes solutions have been constructed by searching for, or postulating, the Lagrangian function  $L(F)$  in the framework of F–P dual formalism [21].<sup>3</sup> As already mentioned in the Introduction, here we will construct a family of spherically symmetric, static, charged regular black hole metrics by imposing three conditions: (a) the weak energy condition should be satisfied, (b) the energy–momentum tensor should have the symmetry  $T^0_0 = T^1_1$ , and (c) these metrics have to asymptotically behave as the Reissner–Nordström black hole metric. Therefore, in this section we will derive the equations which will be used in this construction, from the WEC.

For simplicity and future convenience, we replace the variable  $r$  that appears in Eqs. (8) and (9) with a new variable  $x$  which is defined as  $r = 1/x$  and obviously  $x \in [0, \infty)$ . It is evident that by employing the new variable  $x$ , the WEC inequalities, namely Eqs. (8) and (9), now read

$$x^4 \frac{dm(x)}{dx} \leq 0 \quad (10)$$

and

$$4x^3 \frac{dm(x)}{dx} + x^4 \frac{d^2m(x)}{dx^2} \leq 0. \quad (11)$$

It should be noted that these conditions plus regularity imply that the regular black hole metric function must satisfy the following limit

$$-\frac{dm(x)}{dx} x^4 \rightarrow b \quad \text{when } x \rightarrow \infty \quad (12)$$

where  $b$  is a positive constant.

At this point, we should stress that if we want the regular black hole metric to behave asymptotically as the Schwarzschild black hole, then we have to demand our metric function to satisfy the following condition

$$m(x) \neq 0 \quad \text{when } x \rightarrow 0. \quad (13)$$

However, if we want the regular black hole metric to behave asymptotically as the Reissner–Nordström black hole, then we must also require our metric function to satisfy the condition

$$-\frac{dm(x)}{dx} \neq 0 \quad \text{when } x \rightarrow 0. \quad (14)$$

<sup>3</sup> The F–P dual formalism is briefly presented in Appendix A.

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