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Spectral action and gravitational effects at the Planck scale

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ABSTRACT

We discuss the possibility to extend the spectral action up to energy close to the Planck scale, taking also into account the gravitational effects given by graviton exchange. Including this contribution in the theory, the coupling constant unification is not compromised but is shifted to the Planck scale rendering all gauge couplings asymptotically free. In the scheme of noncommutative geometry, the gravitational effects change the main standard model coupling constants, leading to a restriction of the free parameters of the theory compatible with the Higgs and top mass prediction. We also discuss consequences for the neutrino mass and the see-saw mechanism.

1. Introduction

Noncommutative geometry [1–4] allows to handle a large variety of geometrical frameworks from a totally algebraic point of view. In particular it is very useful in the derivation of models in high energy physics, such as the Yang–Mills gauge theories [5–9]. In the current state the noncommutative geometry structure of gauge theories is understood to be an "almost commutative" geometry, i.e. the product of continuous geometry, representing space–time, times an internal algebra of finite dimensional matrices. In this geometric framework the spectral action principle [10] enables the retrieval of the full standard model of high energy physics, including the Higgs field: the standard model is put on the same footing as geometrical general relativity, making it possible the unification with gravity. In fact, the application of noncommutative geometry to gauge theories of strong and electroweak forces is a very original way to fully geometrize the interaction of elementary particles. Furthermore, it has been shown [11] that it is possible to extend the standard model by including an additional singlet scalar field that stabilizes the running coupling constants of the Higgs field. This singlet scalar field is closely related to the right-handed Majorana neutrinos, conferring them mass, and leading to the prediction of the seesaw mechanism which explains the large difference between the masses of neutrinos and those of the other fermions. A recent model [12] shows the possibility of a further extension, going one step higher in the construction of the noncommutative manifold, in a sort of noncommutative geometry grand unification: here it is pointed out that there could be a "next level" in noncommutative geometry, intertwined with the Riemannian and spin structure of space–time, where the singlet-scalar field arises. Accordingly, it naturally appears at high scale, near to the Planck scale.

A possible framework for describing interactions at energies and momenta below the Planck scale is given in [13,14]. In this Letter we check the possibility to extend the unification scale up to the Planck scale $M_P \equiv \sqrt{\hbar c/G_N} \simeq 10^{19}$ GeV, including not negligible gravitational effects. For a theory dealing with the unification of gauge theory and gravity, a more natural scale is the Planck scale. The usual strategy is to use the spectral action as an effective action at a fixed scale, of the order of the unification scale, and to impose the additional relations between the independent parameters of the standard model. Then, using the renormalization group (RG) equations, one can let these parameters run to their value at low scales and evaluate the Higgs, the top and neutrino masses. The question here is: what is the predictive power of this extended model with exchange of gravitons at the Planck scale? We want to see how the gravitational effects change the main running coupling constants and if they lead to a restriction on the free parameters of the theory still compatible with the Higgs, top and neutrino mass predictions.

In [15] Marcolli and Estrada carried out a similar analysis within the asymptotic safety scenario with Gaussian matter fixed point; differently from this Letter, they have not considered the effect of the scalar field σ introduced in [11], which is necessary in order to reproduce the seesaw mechanism and to have the Higgs mass with its correct value.

This Letter is organized as follows. In Section 2, some ingredients and the main results of the spectral action principle are shown: the derivation of the full standard model bosonic action plus the singlet scalar field and gravity. In Section 3, the

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gravitational contributions to the three gauge couplings, not negligible at the Planck scale, are presented. In Section 4, it is shown how the gravitational effects change the RG equations of the Yukawa and self-interaction Higgs couplings leading to a restriction of the free parameters of the theory compatible with the Higgs and top mass. The final section contains conclusions and some comments.

2. The spectral action

We recall the main features of the spectral action, referring to the original works [1,10] for the full treatment. Those familiar with this calculation can skip to the next section.

The basic ingredients of noncommutative geometry are: an algebra $\mathcal{A}$, which involves the topology of space–time and its noncommutative generalization, a Hilbert space $\mathcal{H}$ on which the algebra acts, containing the fermionic degrees of freedom, and a generalized Dirac operator D which encodes the metric structure of the space. These three objects form the so called spectral triple. The triple is said to be even if there is an operator Γ on $\mathcal{H}$ such that $\Gamma=\Gamma^*$, $\Gamma^2=1$ and

$$\Gamma D + D\Gamma = 0; \qquad \Gamma a - a\Gamma = 0, \quad \forall a \in \mathcal{A}. \tag{2.1}$$

A spectral triple, enlarged with an anti-unitary operator J on $\mathcal{H}$ that obeys to: 1) $J^2=\pm\mathbb{I}$; 2) $JD=\pm DJ$; 3) $J\Gamma=\pm\Gamma J$ (with choice of signs dictated by the KO-dimension of the spectral triple), is said to be real. A real even spectral triple defines a gauge theory, with the gauge fields arising like the inner fluctuations of the Dirac operator

$$D_A = D + A + JAJ \tag{2.2}$$

where A is the one form connection given by the commutator of the Dirac operator D and the elements of the algebra, $A=\sum_i a_i[D,b_i]$; the Dirac operator is the product of a continuous part representing space–time, times an internal part of finite dimensional matrices:

$$D = \partial\!\!\!/_\omega \otimes \mathbb{I}_F + \gamma^5 \otimes D_F \tag{2.3}$$

where $\partial\!\!\!/_\omega \equiv \gamma^\mu(\partial_\mu + \omega_\mu)$ and

$$D_F = \begin{pmatrix} 0 & \mathcal{M} & \mathcal{M}_R & 0 \\ \mathcal{M}^\dagger & 0 & 0 & 0 \\ \mathcal{M}_R^\dagger & 0 & 0 & \mathcal{M}^* \\ 0 & 0 & \mathcal{M}^T & 0 \end{pmatrix},$$

$$\text{with } \mathcal{M} = \begin{pmatrix} M_l & 0 \\ 0 & M_q \end{pmatrix}, \ \mathcal{M}_R = \begin{pmatrix} M_R & 0 \\ 0 & 0 \end{pmatrix}. \tag{2.4}$$

The matrices $\mathcal{M}$ and $\mathcal{M}_R$, via M_l, M_q, and M_R, contain respectively Dirac and Majorana masses, or better Yukawa couplings of leptons, quark and Majorana neutrinos.

From the Dirac operator D_A, we can deduce the full bosonic action of high energy physics coupled to gravity [7, Sect. 4.1] through the regularization of its eigenvalues,

$$S_B[A] \equiv \operatorname{Tr} f\left(\frac{D_A^2}{\Lambda^2}\right) \tag{2.5}$$

where f is a smooth cut-off function and Λ is the cut-off scale of the order of the unification scale. The parameter Λ is used to obtain an asymptotic series for the spectral action via the heath kernel expansion; the physically relevant terms appear with a non-negative power of Λ as coefficient. One could show that this bosonic action is derivable from its fermionic counterpart via the renormalization flow in presence of anomalies [16–18]. The fermionic action is given by

$$S_F = \overline{J\psi}(D + A + JAJ)\psi. \tag{2.6}$$

Let us see the form of the action starting from the formula for a second-order elliptic differential operator D_A^2 of the form

$$D_A^2 = -\left(g^{\mu\nu}\partial_\mu\partial_\nu + K^\mu\partial_\mu + L\right). \tag{2.7}$$

This operator can be written using a connection ∇_μ so that

$$D_A^2 = -\left(g^{\mu\nu}\nabla_\mu\nabla_\nu + E\right). \tag{2.8}$$

Explicitly, $\nabla_\mu = \nabla_\mu^{[R]} + \omega_\mu$ contains both Riemann $\nabla_\mu^{[R]}$ and “gauge” ω parts, with

$$\omega_\mu = \frac{1}{2}g_{\mu\nu}\left(K^\nu + g^{\rho\sigma}\Gamma^\nu_{\rho\sigma}\right). \tag{2.9}$$

Using this ω_μ and L, we find E and compute the curvature $\Omega_{\mu\nu}$ of ∇:

$$\begin{aligned} &E \equiv L - g^{\mu\nu}\partial_\nu(\omega_\mu) - g^{\mu\nu}\omega_\mu\omega_\nu + g^{\mu\nu}\omega_\rho\Gamma^\rho_{\mu\nu}; \\ &\Omega_{\mu\nu} \equiv \partial_\mu(\omega_\nu) - \partial_\nu(\omega_\mu) - [\omega_\mu, \omega_\nu]. \end{aligned} \tag{2.10}$$

The spectral action has a heath kernel expansion in a power series in terms of Λ^{-1} as

$$\begin{aligned} \operatorname{Tr} f\left(\frac{D_A^2}{\Lambda^2}\right) = {}& 2\Lambda^4 f_0 a_0\left(D_A^2\right) + 2\Lambda^2 f_2 a_2\left(D_A^2\right) + f_4 a_4\left(D_A^2\right) \\ & + O\left(\Lambda^{-2}\right), \end{aligned} \tag{2.11}$$

where the f_k are momenta of the function f,

$$f_0 = \int_0^\infty u f(u)\,du, \qquad f_2 = \int_0^\infty f(u)\,du, \qquad f_{2n+4} = (-)^n \partial_u^n f(u) \tag{2.12}$$

and the coefficients $a_n(x,P)$ are called the Seeley–DeWitt coefficients [19,20]. They are equal to zero for n odd and the first three even coefficients are given by

$$\begin{aligned} a_0(x,P) &= (4\pi)^{-m/2}\operatorname{Tr}(\mathbb{I}), \\ a_2(x,P) &= (4\pi)^{-m/2}\operatorname{Tr}(-R/6 + E), \\ a_4(x,P) &= (4\pi)^{-m/2}\operatorname{Tr}\big(-12R^{\mu}_{;\mu} + 5R^2 - 12R_{\mu\nu}R^{\mu\nu} - 60RE \\ &\quad + 180E^2 + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} + 60E^{\mu}_{;\mu} + 30\Omega_{\mu\nu}\Omega^{\mu\nu}\big). \end{aligned} \tag{2.13}$$

3. Higgs-singlet scalar potential and gravity

By inserting the relations for the Seeley–DeWitt coefficients (2.13) into (2.11) we obtain the standard model action plus a new singlet scalar field coupled to gravity [21, Eq. (5.49)]:

$$\begin{aligned} S_B = {}& \frac{24}{\pi^2}f_4\Lambda^4\int d^4x\sqrt{g} - \frac{2}{\pi^2}f_2\Lambda^2 \\ & \times \int d^4x\sqrt{g}\left[R + \frac{1}{2}a\overline{H}H + \frac{1}{4}c\sigma^2\right] \\ & + \frac{1}{2\pi^2}f_0\int d^4x\sqrt{g}\bigg[\frac{1}{30}\left(-18C^2_{\mu\nu\rho\sigma} + 11R^*R^*\right) \\ & + \frac{5}{3}g_1^2B_{\mu\nu}^2 + g_2^2\mathbf{W}_{\mu\nu}^2 + g_3^2\mathbf{V}_{\mu\nu}^2 + \frac{1}{6}aR\overline{H}H + b(\overline{H}H)^2 \\ & + a(\nabla_\mu H)^2 + 2e\overline{H}H\sigma^2 + \frac{1}{2}d\sigma^4 + \frac{1}{12}cR\sigma^2 + \frac{1}{2}c(\partial_\mu\sigma)^2\bigg] \end{aligned} \tag{3.1}$$