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On chromoelectric (super)conductivity of the Yang–Mills vacuum

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ABSTRACT

We argue that in the Copenhagen ("spaghetti") picture of the QCD vacuum the chromomagnetic flux tubes exhibit chromoelectric superconductivity. We show that the superconducting chromoelectric currents in the tubes may be induced by the topological charge density.

The nonperturbative structure of the ground state of QCD vacuum is one of the most interesting unsolved problems in quantum field theory. At zero temperature the ground state exhibits a mass gap, breaks chiral symmetry and supports confinement of color sources, quarks and gluons. The confining properties of the QCD ground state were intensively studied last decades resulting in a number of phenomenological approaches to this problem.

One of the popular approaches is the "spaghetti vacuum" picture (the Copenhagen vacuum): the QCD vacuum is considered to be populated by evolving vortex tubes which carry a chromomagnetic flux [1–5]. An isolated color charge – for example, a quark – scatters off the vortices and develops an infinitely large free energy. As a result, the quarks may appear in the vacuum only in a form of colorless (hadronic) states bounded by a chromoelectric string [2].

The standard mechanism of formation of the chromomagnetic vortices is as follows. The perturbative vacuum of QCD – which is paramagnetic due to the asymptotic freedom – has an unstable mode towards formation of a chromomagnetic field [6]. However, in the background of a homogeneous chromomagnetic field the gluon part of the vacuum energy develops an imaginary part to large chromomagnetic moment of the gluon [1]. This implies that the homogeneous chromomagnetic field is also unstable towards squeezing of the chromomagnetic field into separate parallel flux tubes (vortices) [4], similarly to the Abrikosov vortex lattice in a mixed state of an ordinary type-II superconductor in an external magnetic field [7]. Finally, due to global rotational and Lorentz invariance of the QCD vacuum, the chromomagnetic field has locally a domain-like structure [3]: the field has different orientation in different domains. Due to the fact that the vortices follow the orientation of the chromomagnetic field, the vortex lines form an intertwining entangled structure, hence the name "spaghetti".

Thus, the Copenhagen confining mechanism has a tight relation to ideas from ordinary superconductivity such as condensation and flux tube (vortex) formation. However, in addition to the mentioned features there exists another, primary phenomenon which is associated with ordinary superconductivity which is the superconductivity itself (i.e., the perfect conductivity of an electric current). In this Letter we would like to show that the Copenhagen vacuum is not just "analogous" to an ordinary superconductivity: in this picture the Copenhagen vacuum *is* a chromoelectric superconductor from the point of view of the transport properties.

Why the Copenhagen vacuum should be a chromoelectric superconductor? A simple answer is because in this picture the chromomagnetic tubes are formed due to the gluon condensate while the gluons are carrying a color charge. The condensation of the color charges should lead, naively, to the (chromo)superconducting phenomenon. However, our considerations may contain a caveat: in the ordinary superconductivity the Cooper-pair condensate has a macroscopic order over large distances and this property is the core reason why the electric current may be transported by the uniform condensate without dissipation. On the contrary, the QCD vacuum in the Copenhagen picture has a domain-like structure with each domain possessing its own orientation (both in

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color and coordinate spaces) of the gluon condensate so that the long-range order is absent. Nevertheless, we argue below that this property is not an obstacle due to the long-range order which is maintained *along* the chromomagnetic vortices. We arrive to the picture that in the spaghetti vacuum the chromoelectric current should be able to stream without dissipation along the chromomagnetic tubes. Basically, the chromomagnetic tubes work like specific, hollow wires which are able to carry the chromoelectric current without resistance.

As one of the possible consequences of the color superconductivity one can expect probe quarks to propagate along the flux tubes over arbitrary distances, so that the tubes can be considered as "fermionic guides" [8–11]. From the phenomenological perspective the long-range propagation of quarks may lead to the phenomenon of chiral superfluidity of the quark–gluon plasma [12].

The Yang–Mills Lagrangian is

$$\mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu}, \tag{1}$$

where $F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g\epsilon^{abc} A^b_\mu A^c_\nu$ is the strength tensor of the $SU(2)$ gluon field A^a_μ.

For simplicity, we consider the $SU(2)$ gauge field instead of more phenomenologically relevant $SU(3)$ fields since the latter solutions may be obtained – following the general construction of Ambjorn–Olesen [4] – by an imbedding the $SU(2)$ solutions into the $SU(3)$ color group.

The corresponding equations of motion are as follows

$$\partial^\mu F^a_{\mu\nu} + g\epsilon^{abc} A^{b\mu} F^c_{\mu\nu} = 0. \tag{2}$$

Following Ambjorn and Olesen [4] we consider the state of the Yang–Mills theory in a uniform chromomagnetic field directed along the third spatial axis. In the color space the chromomagnetic field is assumed to be directed in the third axis as well:

$$F^{a,\mathrm{ext}}_{\mu\nu} \sim \delta^{a,3}(\delta_{\mu 1}\delta_{\nu 2} - \delta_{\mu 2}\delta_{\nu 1}). \tag{3}$$

In oder to obtain such a configuration, one can add a homogeneous Abelian magnetic flux in third direction in color space [4]:

$$A^3_1 = -x_1 \frac{B}{2}, \qquad A^3_2 = x_2 \frac{B}{2}. \tag{4}$$

For definiteness we take $B > 0$.

The ground state solution to the equations of motion (2) has certain remarkable properties. The solution is a function of the transverse – with respect to the spatial direction of the external chromomagnetic field (3) – coordinates $x^\perp = (x_1, x_2)$ and it is independent on the longitudinal coordinates $x^\parallel = (x_0, x_3)$.

The longitudinal components of the vector fields are vanishing in the ground state, $A^a_0 = A^a_3 = 0$, so that the equations of motion (2) involve only the transversal components A^a_i with $i = 1, 2$. The latter can conveniently be rewritten in the complex notations by introducing the following combinations for all vector fields $\mathcal{O}_i$ with $i = 1, 2$: $\mathcal{O} = \mathcal{O}_1 + i\mathcal{O}_2$ and $\bar{\mathcal{O}} = \mathcal{O}_1 - i\mathcal{O}_2$. These relations imply $\bar{\mathcal{O}} = \mathcal{O}^*$ for all real vector fields $\mathcal{O}_i$. Defining the complex coordinate $z = x_1 + ix_2$ and complex derivative $\partial = \partial_1 + i\partial_2$, we find the non-canonical relations $\bar\partial z = \partial \bar z = 2$ and $\partial z = \bar\partial \bar z = 0$.

The off-diagonal gluonic fields $A^{1,2}_\mu$ can be combined into two complex-valued fields:

$$A^\pm_\mu = \frac{1}{\sqrt{2}}\left(A^1_\mu \mp i A^2_\mu\right). \tag{5}$$

These combinations are not independent, $A^\pm_\mu \equiv (A^\mp_\mu)^\dagger$, so that below we will work with the A^-_μ field only.

The ground state can be described by two complex functions $A = A(x^\perp)$ and $A^3 = A^3(x^\perp)$ with

$$A \equiv A^- = A^-_1 + iA^-_2, \tag{6}$$

$$A^3 = A^3_1 + iA^3_2, \tag{7}$$

and their complex conjugates. The combinations (6) and (7) correspond to, respectively, the off-diagonal and diagonal components of the A^a_i fields. The color direction is defined by the background chromomagnetic field. The alternative (barred) combination of the off-diagonal A fields is zero in the ground state, $\bar A^- = A^-_1 - iA^-_2 = 0$. Notice that $A^+ \equiv (\bar A^-)^* = 0$ and $\bar A^+ = (A^-)^*$.

The constraints (3) for $a = 1, 2$ can now be rewritten as a single complex equation:

$$\bar\partial A = -\frac{gB}{2}\bar z A, \tag{8}$$

which is well known from the work of Abrikosov [7] to possess finite-energy solutions with a lattice symmetry.

The solution for this equation minimizing the remaining terms contributing to the energy integrated over the transversal plane

$$E_\perp = \int \frac{1}{2}\left(F^3_{12}\right)^2 = \int \frac{1}{2}\left(B - \frac{g}{2}|A|^2\right)^2, \tag{9}$$

was constructed in terms of θ-functions. In the background of the strong chromomagnetic field the vacuum structure resembles the Abrikosov lattice in the mixed phase of the type-II superconductors [7]. In analogy with the lattice of the Abrikosov vortices in a superconductor, the chromomagnetic field in Yang–Mills theory organizes itself in similar periodic structures [4].

The ground state solution by Ambjorn and Olesen is given [up to a gauge factor due to a different parametrization of magnetic field (4)] by the following formula [4]:

$$A(x_1, x_2) = \phi_0 e^{igBx_2 \frac{x_1 + ix_2}{2}} \theta_3\left(\frac{(x_1 + ix_2)\nu}{L_B}, e^{\frac{2i\pi}{3}}\right), \tag{10}$$

$$\nu = \frac{\sqrt[4]{3}}{\sqrt{2}}, \qquad L_B = \sqrt{\frac{2\pi}{gB}}, \tag{11}$$

where θ_3 is the third Jacobi theta function, and the overall factor $\phi_0 \approx 2.9\sqrt{B/g}$ is determined numerically by a minimization of the energy functional (9).

The global energy minimum is reached for the equilateral triangular lattice (which is also called the hexagonal lattice) solutions of Eq. (8). Another local minimum is found for a square lattice.

The geometrical pattern of the lattice structure in the Yang–Mills theory is determined by the Abrikosov ratio,

$$\beta_\mathrm{A} = \mathrm{Area}_\perp \left(\int dx^2_\perp |A|^4\right) \Big/ \left(\int dx^2_\perp |A|^2\right)^2, \tag{12}$$

which can be expressed in terms of generalized θ-functions [4]. The global minimum of the energy functional (9) is

$$E_{\perp,\min} = \mathrm{Area}_\perp \frac{B^2}{2}\left(1 - \frac{1}{\beta_\mathrm{A}}\right), \tag{13}$$

where for the hexagonal structure the Abrikosov ratio is $\beta_\mathrm{A} \approx 1.16$ similarly to an ordinary type-II superconductor [13].

It is worth noting that even in models where the forth-order interaction terms are more complicated and, as a consequence, another definition of β_A is needed, one still finds that the global energy minimum still corresponds to the hexagonal lattice pattern [13,14,16,17].

The gluon field (10) is shown in Fig. 1. The chromomagnetic vortices are arranged in the hexagonal structure. In the center of each vortex the gluon field (6) is vanishing and the phase of this field winds by the angle 2π, similarly to the usual Abrikosov