Physics Letters B 730 (2014) 70–75

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Energy and transverse momentum fluctuations in the equilibrium quantum systems

M.I. Gorenstein [a,b,*], M. Rybczyński [c]

[a] Bogolyubov Institute for Theoretical Physics, Kiev, Ukraine
[b] Frankfurt Institute for Advanced Studies, Frankfurt, Germany
[c] Institute of Physics, Jan Kochanowski University, Kielce, Poland

ARTICLE INFO

Article history:
Received 7 August 2013
Accepted 14 January 2014
Available online 18 January 2014
Editor: W. Haxton

Keywords:
Statistical models
Event-by-event fluctuations
Nucleus–nucleus collisions

ABSTRACT

The fluctuations in the ideal quantum gases are studied using the strongly intensive measures $\Delta[A,B]$ and $\Sigma[A,B]$ defined in terms of two extensive quantities A and B. In the present Letter, these extensive quantities are taken as the motional variable, $A = X$, the system energy E or transverse momentum P_T, and number of particles, $B = N$. This choice is most often considered in studying the event-by-event fluctuations and correlations in high energy nucleus–nucleus collisions. The recently proposed special normalization ensures that Δ and Σ are dimensionless and equal to unity for fluctuations given by the independent particle model. In statistical mechanics, the grand canonical ensemble formulation within the Boltzmann approximation gives an example of independent particle model. Our results demonstrate the effects due to the Bose and Fermi statistics. Estimates of the effects of quantum statistics in the hadron gas at temperatures and chemical potentials typical for thermal models of hadron production in high energy collisions are presented. In the case of massless particles and zero chemical potential the Δ and Σ measures are calculated analytically.

1. Introduction

Experimental and theoretical studies of the event-by-event (e-by-e) fluctuations in nucleus–nucleus (A+A) collisions give new information about properties of the strongly interacting matter and its phases. A possibility to observe signatures of the QCD matter critical point inspired the energy and system size scan program of the NA61/SHINE Collaboration at the SPS CERN [1] and the low energy scan program of the STAR and PHENIX Collaborations at the RHIC BNL [2]. In these studies one measures and then compares the e-by-e fluctuations in collisions of different nuclei at different collision energies. The average sizes of the created physical systems and their e-by-e fluctuations are expected to be rather different [3]. This strongly affects the observed hadron fluctuations, i.e. the measured quantities would not describe the local physical properties of the system but rather reflect the system size fluctuations. For instance, A + A collisions with different centralities may produce a system with approximately the same local properties (e.g., the same temperature and baryonic chemical potential) but with the volume changing significantly from interaction to interaction. Note that in high energy collisions the average volume of created matter and its variations from collision to collision are usually out of experimental control (i.e. these volume variations are difficult or even impossible to measure).

In the statistical mechanics the extensive quantity A is proportional to the system volume V, whereas intensive quantity has a fixed finite value in the thermodynamical limit $V \to \infty$. The intensive quantities are used to describe the local properties of a physical system. In particular, an equation of state of the matter is usually formulated in terms of the intensive physical quantities, e.g., the pressure is considered as a function of temperature and chemical potentials. In the statistical systems outside of the phase transition regions, a mean value of fluctuating extensive quantity, $\langle A\rangle$, and its variance, $\mathrm{Var}(A) = \langle A^2\rangle - \langle A\rangle^2$, are both proportional to the volume V in the limit of large volumes. The scaled variance,

$$\omega[A] = \frac{\langle A^2\rangle - \langle A\rangle^2}{\langle A\rangle}, \qquad (1)$$

is therefore an intensive quantity. However, the scaled variance being an intensive quantity depends on the system size fluctuations.

Strongly intensive quantities introduced in Ref. [4] are independent of the average volume and of volume fluctuations. These quantities were suggested for and are used in studies of e-by-e fluctuations of hadron production in A + A collisions. Strongly intensive measures of fluctuations are defined in terms of two arbitrary extensive quantities A and B. In the present study we

* Corresponding author.

http://dx.doi.org/10.1016/j.physletb.2014.01.028
0370-2693

consider a pair of extensive variables – the motional extensive variable $X = x_1 + \cdots + x_N$ as a sum of single particle variables x_j, with $j = 1, \ldots, N$, and number of particles N. These measures were recently studied within the UrQMD simulations in Ref. [5]. The case of two hadron multiplicities A and B in A + A collisions has been considered within the HSD transport model in Ref. [6]. At the beginning we identify a single particle variable x with the particle energy ϵ and then consider the particle transverse momentum p_T.

The strongly intensive measure $\Delta[X,N]$ and $\Sigma[X,N]$ are defined as [4]:

$$\Delta[X,N] = \frac{1}{C_\Delta}\big[\langle N\rangle\omega[X] - \langle X\rangle\omega[N]\big], \tag{2}$$

$$\Sigma[X,N] = \frac{1}{C_\Sigma}\big[\langle N\rangle\omega[X] + \langle X\rangle\omega[N] - 2\big(\langle XN\rangle - \langle X\rangle\langle N\rangle\big)\big], \tag{3}$$

where C_Δ and C_Σ are the normalization factors, and the scaled variances $\omega[X]$ and $\omega[N]$ are given by Eq. (1).

In Ref. [7] a special normalization for the strongly intensive measures Δ and Σ has been proposed. Namely, the properly normalized strongly intensive quantities assume the value *one* for fluctuations given by the independent particle model (IPM). For the X and N extensive quantities the proposed normalization reads [7]:

$$C_\Delta = C_\Sigma = \omega[x]\cdot\langle N\rangle, \quad \omega[x] \equiv \frac{\overline{x^2} - \bar{x}^2}{\bar{x}}. \tag{4}$$

Note that the overline denotes averaging over a single particle inclusive distribution, whereas $\langle\cdots\rangle$ represents averaging over multiparticle states of the system.

The first strongly intensive measure for fluctuations, the so-called Φ measure, was introduced a long time ago in Ref. [8]. The Φ quantity for the ideal quantum gases was considered in Ref. [9]. There were numerous attempts to use the Φ measure describing fluctuations in experimental data [10] and models [11]. In general, however, Φ is a dimensional quantity and it does not have a characteristic scale for a quantitative analysis of e-by-e fluctuations for different observables. Note that the latter properties were clearly disturbing. The Φ measure can be expressed in terms of Σ [4]. A presence of additional fluctuation measure Δ and utilization of special normalization conditions for both Δ and Σ give essential advantages in application to the data analysis in A + A collisions.

In the present Letter we study the strongly intensive measures (2) and (3) with normalization factors (4) for the relativistic ideal quantum gases in the grand canonical ensemble. The Letter is organized as follows. In Section 2 we calculate the $\Delta[X,N]$ and $\Sigma[X,N]$ quantities for the ideal quantum gases in the grand canonical ensemble. Analytical and numerical results suitable for the hadron gas created in A + A collisions are presented in Section 3. A summary in Section 4 closes the article.

2. Ideal quantum gas

The grand canonical ensemble (GCE) partition function reads:

$$\Xi(V,T,\lambda) = \sum_N \sum_\alpha \lambda^N \exp(-\beta E_\alpha), \tag{5}$$

where V is the system volume, $\beta \equiv T^{-1}$ is the inverse system temperature, $\lambda \equiv \exp(\beta\mu)$ denotes the fugacity and μ the chemical potential. The index α numerates the system quantum states, and N is the number of particles. The ensemble average values of the kth moments ($k = 1, 2, \ldots$) of any state quantity A are calculated as:

$$\langle A^k\rangle = \frac{1}{\Xi}\sum_N\sum_\alpha A^k\lambda^N\exp(-\beta E_\alpha). \tag{6}$$

The GCE partition function (5) can be presented in the form

$$\Xi = \exp\left\{V\eta^{-1}d\int\frac{d^3p}{(2\pi)^3}\ln\big[1+\eta\lambda\exp(-\beta\epsilon)\big]\right\}, \tag{7}$$

where d is the number of particle internal degrees of freedom and $\epsilon \equiv \sqrt{m^2 + \mathbf{p}^2}$ is the particle energy with m being the particle mass and $\mathbf{p}$ its momentum. The values $\eta = -1$ and $\eta = 1$ correspond to the Bose and Fermi statistics, respectively, whereas $\eta = 0$ to the Boltzmann approximation. Using the presentation (7) one can calculate the averages (6) for the 1st and 2nd moments of the energy E and number of particles N:

$$\langle N\rangle = \frac{1}{\Xi}\lambda\frac{\partial\Xi}{\partial\lambda} = V\rho,$$

$$\rho \equiv d\int\frac{d^3p}{(2\pi)^3}\frac{1}{\lambda^{-1}\exp(\epsilon/T)+\eta}, \tag{8}$$

$$\langle N^2\rangle = \frac{1}{\Xi}\left(\lambda\frac{\partial}{\partial\lambda}\right)^2\Xi = V^2\rho^2 + VI_N,$$

$$I_N \equiv d\int\frac{d^3p}{(2\pi)^3}\frac{\lambda^{-1}\exp(\epsilon/T)}{[\lambda^{-1}\exp(\epsilon/T)+\eta]^2}, \tag{9}$$

$$\langle E\rangle = -\frac{1}{\Xi}\frac{\partial\Xi}{\partial\beta} = V\varepsilon,$$

$$\varepsilon \equiv d\int\frac{d^3p}{(2\pi)^3}\frac{\epsilon}{\lambda^{-1}\exp(\epsilon/T)+\eta}, \tag{10}$$

$$\langle E^2\rangle = \frac{1}{\Xi}\frac{\partial^2}{\partial\beta^2}\Xi = \langle E\rangle^2 + VI_E,$$

$$I_E \equiv d\int\frac{d^3p}{(2\pi)^3}\frac{\epsilon^2\lambda^{-1}\exp(\epsilon/T)}{[\lambda^{-1}\exp(\epsilon/T)+\eta]^2}, \tag{11}$$

$$\langle EN\rangle = -\frac{1}{\Xi}\frac{\partial}{\partial\beta}\lambda\frac{\partial}{\partial\lambda}\Xi = \langle N\rangle\langle E\rangle + VI_{EN},$$

$$I_{EN} \equiv d\int\frac{d^3p}{(2\pi)^3}\frac{\epsilon\lambda^{-1}\exp(\epsilon/T)}{[\lambda^{-1}\exp(\epsilon/T)+\eta]^2}, \tag{12}$$

where $\rho \equiv \langle N\rangle/V$ and $\varepsilon \equiv \langle E\rangle/V$ denote the particle number density and the energy density, respectively.

From Eqs. (8)–(12) one finds for the scaled variances:

$$\omega[N] \equiv \frac{\langle N^2\rangle - \langle N\rangle^2}{\langle N\rangle} = \frac{I_N}{\rho}, \qquad \omega[E] \equiv \frac{\langle E^2\rangle - \langle E\rangle^2}{\langle E\rangle} = \frac{I_E}{\varepsilon}. \tag{13}$$

They describe the fluctuations of the number of particles and the system energy at fixed volume V. The scaled variances in Eq. (13) are intensive quantities, they depend only on T and μ. The quantities (13) are independent of the particle degeneracy factor d. Note that there is a (positive) correlation between the energy E and particle number N:

$$\langle EN\rangle - \langle E\rangle\langle N\rangle = VI_{EN} > 0. \tag{14}$$

The moments of single particle energy ϵ ($k = 1, 2$) are

$$\overline{\epsilon^k} = \frac{d}{\rho}\int\frac{d^3p}{(2\pi)^3}\frac{\epsilon^k}{\lambda^{-1}\exp(\epsilon/T)+\eta}, \tag{15}$$

and the scaled variance $\omega[\epsilon]$ is

$$\omega[\epsilon] = \frac{\overline{\epsilon^2} - \bar{\epsilon}^2}{\bar{\epsilon}}. \tag{16}$$