Physics Letters B 730 (2014) 226–230

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Twist free energy and critical behavior of 3D $U(1)$ LGT at finite temperature

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ARTICLE INFO

Article history:
Received 9 November 2013
Received in revised form 23 January 2014
Accepted 27 January 2014
Available online 30 January 2014
Editor: L. Alvarez-Gaumé

Keywords:
Abelian gauge model
Phase diagram
Universality
Critical exponents
Deconfinement phase transition

ABSTRACT

The twist free energy is computed in the Villain formulation of the 3D $U(1)$ lattice gauge theory at finite temperature. This enables us to obtain renormalization group equations describing a critical behavior of the model in the vicinity of the deconfinement phase transition. These equations are used to check the validity of the Svetitsky–Yaffe conjecture regarding the critical behavior of the lattice $U(1)$ model. In particular, we calculate the two-point correlation function of the Polyakov loops and determine some critical indices.

1. Introduction

The critical behavior of pure lattice gauge theories (LGTs) at finite temperatures is well understood for non-abelian $SU(N)$ theories in various dimensions. In particular, the phase structure of a finite-temperature three-dimensional (3D) pure $SU(N)$ LGT with the standard Wilson action is thoroughly investigated both for $N = 2, 3$ and for the large-N limit (see, *e.g.*, [1] and references therein). The transition is second order for $N = 2, 3$ and first order for $N > 4$. In the case of the $SU(4)$ gauge group, most works agree that the transition is weakly first order. The deconfining transition in $SU(N = 2, 3)$ LGTs belongs to the universality class of 2D $Z(N = 2, 3)$ Potts models. All these phase transitions are characterized by the spontaneous symmetry breaking of a $Z(N)$ global symmetry of the lattice action in the high-temperature deconfining phase.

Surprisingly, the situation is much less clear for the 3D $U(1)$ LGT. The present state of affairs can be briefly summarized as follows. 3D theory was studied by Parga using Lagrangian formulation of the theory [2]. At high temperatures the system becomes effectively two-dimensional, in particular the monopoles of the original $U(1)$ gauge theory become vortices of the 2D system. The partition function turns out to coincide (in the leading order of the high-temperature expansion) with the 2D XY model in the Villain representation. The XY model is known to have the Berezinskii–Kosterlitz–Thouless (BKT) phase transition of the infinite order [3,4]. According to the Svetitsky–Yaffe conjecture the finite-temperature phase transition in the 3D $U(1)$ LGT should belong to the universality class of the 2D XY model [5]. This means, firstly that the global $U(1)$ symmetry cannot be broken spontaneously because of the Mermin–Wagner theorem [6] and, consequently the local order parameter does not exist for this type of the phase transition. Secondly, the correlation function of the Polyakov loops (which become spins of the XY model) decreases with the power law at $\beta \geqslant \beta_c$ implying a logarithmic potential between heavy electrons

$$P(R) \asymp \frac{1}{R^{\eta(T)}}, \tag{1}$$

where the $R \gg 1$ is the distance between test charges. The critical index $\eta(T)$ is known from the renormalization-group analysis of Ref. [4] and equals $\eta(T_c) = 1/4$ at the critical point of the BKT transition. For $\beta < \beta_c$, $t = \beta_c/\beta - 1$ one has

$$P(R) \asymp \exp\bigl[-R/\xi(t)\bigr], \tag{2}$$

where the correlation length $\xi \sim e^{bt^{-\nu}}$ and the critical index $\nu = 1/2$. Therefore, the critical indices η and ν should be the same in the finite-temperature $U(1)$ model if the Svetitsky–Yaffe conjecture holds in this case. The first numerical check of these predictions was performed on the lattices $L^2 \times N_t$ with $L = 16, 32$

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http://dx.doi.org/10.1016/j.physletb.2014.01.055
0370-2693

and $N_t = 4, 6, 8$ in [7]. Though authors of [7] confirm the expected BKT nature of the phase transition, the reported critical index is almost three times larger of that predicted for the XY model, $\eta \approx 0.78$. More recent analytical and numerical studies of Ref. [8] indicate that at least on the anisotropic lattice in the limit of vanishing spatial coupling β_s (where space-like plaquettes are decoupled) the $3D$ $U(1)$ gauge model exhibits the critical behavior similar to the XY spin model. However, numerical simulations of the isotropic model on the lattices up to $L = 256$ and $N_t = 8$ reveal that $\eta \approx 0.49$, *i.e.* still far from the XY value [9]. Thus, so far there is no numerical indications that critical indices of $3D$ $U(1)$ LGT coincide with those of the $2D$ XY model and the question of the universality remains open if β_s is non-vanishing.

On the analytical side one should mention a renormalization group (RG) study of Refs. [5,10]. In both cases a high-temperature and a dilute monopole gas approximations were used for the Villain formulation which helped to derive an effective sine-Gordon model. Resulting RG equations were shown to converge rapidly with iterations to RG equations of the $2D$ XY model. It gives a strong indication that, indeed the nature of the phase transitions in both models is the same. Moreover, since the scaling of the lattice spacing coincides in both cases the critical index ν should also be the same (this however was not proven). Furthermore, neither critical points nor index η has been determined in previous studies.

In this work we re-examine the critical behavior of the Villain formulation of the $3D$ $U(1)$ LGT aiming to compute both critical indices ν and η as well as to determine the location of the critical points. In order to achieve this goal we calculate the free energy of the model in the presence of a twist and express it like a function of a bare coupling, a monopole activity and adimensional ratio of the anisotropic couplings. Varying the lattice cut-off one then finds the RG equations in a standard manner (see, for example, Chapter 4.2.5 in [11]). We analyze the equations thus obtained for different values of N_t. Also, we present results for the correlation function of the Polyakov loops which allow to extract the index η at the critical point.

2. Definition of the model and its dual

We work on a periodic $3D$ lattice $\Lambda = L^2 \times N_t$ with spatial extension L and temporal extension N_t. We introduce anisotropic dimensionless couplings as

$$\beta_t = \frac{1}{g^2 a_t}, \qquad \beta_s = \frac{\xi}{g^2 a_s} = \beta_t \xi^2, \qquad \xi = \frac{a_t}{a_s}, \tag{3}$$

where a_t (a_s) is lattice spacing in the time (space) direction, g^2 is the continuum coupling constant with dimension a^{-1}. $\beta = a_t N_t$ is an inverse temperature.

The compact $3D$ $U(1)$ LGT on the anisotropic lattice in the presence of the twist is defined through its partition function as

$$Z(\beta_t, \beta_s) = \int_0^{2\pi} \prod_{x\in\Lambda} \prod_{n=1}^{3} \frac{d\omega_n(x)}{2\pi} \exp S[\omega + \theta], \tag{4}$$

where S is the Wilson action

$$S[\omega] = \beta_s \sum_{p_s} \cos \omega(p_s) + \beta_t \sum_{p_t} \cos \omega(p_t), \tag{5}$$

$$\omega(p) = \omega_n(x) + \omega_m(x + e_n) - \omega_n(x + e_m) - \omega_m(x) \tag{6}$$

and sums run over all space-like (p_s) and time-like (p_t) plaquettes. We take a constant shift θ_n on a stack of plaquettes wrapping around the lattice in the spatial directions, *e.g.* the shift θ_1 on the plaquettes with coordinates $p = (n_2, n_3; x_1, 0, 0)$ and the shift θ_2 on the plaquettes with coordinates $p = (n_1, n_3; 0, x_2, 0)$ (for a detailed description of the twist in LGT we refer the reader to Ref. [12] where also some properties of the twisted partition function are discussed).

In order to calculate the free energy in the presence of the twist we make the following quite standard steps, proposed first in [4]:

- Perform duality transformations with the twisted partition function.
- Replace the dual Boltzmann weight with the Villain formulation and calculate an effective monopole theory.
- Sum up over monopole configurations in the dilute gas approximation.

All these steps are well known in the context of the $3D$ $U(1)$ LGT and can be easily generalized for the anisotropic lattice in the presence of the twist. For the duality transformations we need an approach of Ref. [13] which takes correctly into account the periodic boundary conditions on the abelian gauge fields. For the anisotropic theory with twist we find

$$Z(\theta_n) = \sum_{h_n=-\infty}^{\infty} e^{i \sum_{n=1}^{2} h_n \theta_n} Z(h_n), \tag{7}$$

where the global summation over h_n enforces the global Bianchi constraint on the periodic system and $Z(h_n)$ is the dual partition function

$$Z(h_n) = \sum_{r(x)=-\infty}^{\infty} \prod_{x} \prod_{n=1}^{3} I_{r(x)-r(x+e_n)+\eta_n(x)}(\beta_n). \tag{8}$$

Here $I_r(x)$ is the modified Bessel function and we have introduced sources $\eta_n(x) = \eta(l)$ as

$$\eta(l) = \begin{cases} h_n, & l \in P_d, \\ 0, & \text{otherwise}, \end{cases} \tag{9}$$

where P_d is a set of links dual to twisted plaquettes (this set forms a closed loop on the dual lattice), $\beta_n = \beta_s$, $n = 3$ and $\beta_n = \beta_t$, $n = 1, 2$. In the limit $\beta_s = 0$ and in the absence of the twist the partition function (7) reduces to ($x = (x_1, x_2)$ runs now over two-dimensional lattice L^2)

$$Z(0) = \sum_{r(x)=-\infty}^{\infty} \prod_{x} \prod_{n=1}^{2} I^{N_t}_{r(x)-r(x+e_n)}(\beta_n). \tag{10}$$

In this limit the model becomes a generalized version of the XY model, and it was studied both analytically and by Monte-Carlo simulations in Ref. [8]. The firm conclusion of Ref. [8] was that the model (10) is in the same universality class as the XY model. Here we are going to study an opposite limit, namely $\beta_t > \beta_s \gg 1$ which lies close to the continuum limit of the full $3D$ $U(1)$ model. When both couplings are large it is customary to use the Villain approximation, *i.e.*

$$I_r(x)/I_0(x) \approx \exp\left(-\frac{1}{2x} r^2\right). \tag{11}$$

The Villain model, obtained by taking the approximation (11) in (7) is generally accepted to have the same universal properties as the original model [2,5]. In the next sections we propose a renormalization group for the Villain model.