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Three-loop field renormalization for scalar field theory with Lorentz violation

Paulo R.S. Carvalho

Departamento de Física, Universidade Federal do Piauí, Campus Ministro Petrônio Portela, 64049-550, Teresina, PI, Brazil

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ABSTRACT

Applying the counterterm method in minimal subtraction scheme we calculate the three-loop quantum correction to field anomalous dimension in a Lorentz-violating O(N) self-interacting scalar field theory. We compute the Feynman diagrams using dimensional regularization and ϵ-expansion techniques. As this approximation corresponds to a three-loop term, to our knowledge this is the first time in literature in which such a loop level is attained for a LV theory.

1. Introduction

In high energy physics the main aspects of many physical effects involving particles and fields such as pair annihilation, Compton effect, positronium lifetime, Bremsstrahlung can be understood by lowest-order perturbative calculations [1,2], although higher-level computations give more precise knowledge about these effects. On the other hand, the many-body behavior of some physical systems is satisfactorily described only if higher-order approximations are used for studying them. As an example, both three-level and one-loop quantum correction for the renormalization group outcome for the correlation function critical exponent η, related to field anomalous dimension, which characterizes a second order phase transition in ferromagnetic systems are null [3,4]. Thus the nonvanishing leading quantum contribution to this critical exponent lies just at two-loop order. As ferromagnetic systems present large thermal fluctuations near critical point, any higher-loop correction, albeit small, is highly relevant for an accurate determination of the numerical value of a critical exponent. For these systems, the critical exponents up to a five-loop level approximation were evaluated [5,6].

All physical phenomena above are described by theories satisfying certain symmetry principles, one of them is Lorentz invariance. However some of these phenomena and many others are been studied in the limit in which this symmetry is violated. These theories were proposed as natural extensions of their Lorentz-invariant (LI) counterparts [7–19]. More specifically, in a recent paper [20], the β function and field anomalous dimension γ were calculated up to two-loop approximation for a Lorentz-violating (LV) O(N) scalar field theory. This theory may have many applications in the standard model LV Higgs sector. The mass in this theory was renormalized up to the same loop level [21]. While the β function and mass were computed up to next-to-leading order, only the leading quantum correction to field anomalous dimension were obtained. The aim of this Letter is to calculate the γ function up to next-to-leading approximation.

We begin this Letter discussing the bare theory for the O(N) scalar field theory with Lorentz violation and its three-loop diagrammatic expansion for two-point function necessary in this work in the Section 2. In the Section 3 we will discuss the evaluation of the three-loop level renormalization constant for field renormalization and the respective loop-order Wilson function γ. We will finalize the Letter in Section 4 with our conclusions.

E-mail address: prscarvalho@ufpi.edu.br.

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2. Basics

2.1. Bare theory

The unrenormalized Euclidean Lagrangian density for the massive self-interacting O(N) LV scalar field theory is given by [20]

$$\mathscr{L} = \frac{1}{2}\partial^{\mu}\phi_B\partial_{\mu}\phi_B + \frac{1}{2}K_{\mu\nu}\partial^{\mu}\phi_B\partial^{\nu}\phi_B + \frac{1}{2}m_B^2\phi_B^2 + \frac{\lambda_B}{4!}\phi_B^4. \tag{1}$$

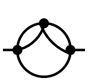
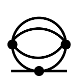
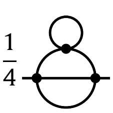
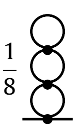
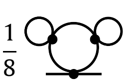
This Lagrangian density is invariant under rotations in an O(N) internal symmetry field space. The field is a N-component vector field and the last term in Eq. (1) represents its quartic self-interaction where $\phi^4 = (\phi_1^2 + \cdots + \phi_N^2)^2$. The quantities ϕ_B, m_B and λ_B are the bare field, mass and coupling constant, respectively. The LV second term above breaks the Lorentz symmetry through the dimensionless symmetric constant coefficients $K_{\mu\nu}$ (the components of $K_{\mu\nu}$ are chosen such that this two-component mathematical object does not transform as a second order tensor under Lorentz transformations) which are the same for all N components of the vector field. This tensor is responsible for a slight symmetry violation when $|K_{\mu\nu}| \ll 1$. We can also see that the unrenormalized inverse free propagator in momentum space of the theory is given by $q^2 + K_{\mu\nu}q^{\mu}q^{\nu} + m_B^2$ and thus we have a modified version of a conventional scalar field theory. Another modification comes from the emergence of the factor

$$\Pi = 1 - \frac{1}{2}K_{\mu\nu}\delta^{\mu\nu} + \frac{1}{8}K_{\mu\nu}K_{\rho\sigma}\delta^{\{\mu\nu}\delta^{\rho\sigma\}} + \cdots \tag{2}$$

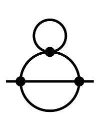
present in the results for the β and γ functions where $\delta^{\{\mu\nu}\delta^{\rho\sigma\}} \equiv \delta^{\mu\nu}\delta^{\rho\sigma} + \delta^{\mu\rho}\delta^{\nu\sigma} + \delta^{\mu\sigma}\delta^{\nu\rho}$. The factor in Eq. (2) has a similar form in Minkowski space–time [20]. These two forms are connected by a Wick rotation when we have $\delta^{\mu\nu} \to \eta^{\mu\nu}$ where $\eta^{\mu\nu}$ is the Minkowski metric tensor. As it is known [6], the bare two-point vertex function $\overline{\Gamma}_B^{(2)}$ has two divergent terms: one proportional to external momentum P^2 and another to bare mass m_B^2. In the process of field renormalization for a scalar field theory, it is needed to renormalize just the former. The latter can be used to mass renormalization purposes. Our task is to analyze the three-loop level field renormalization term for this function. This will be the subject of next section.

2.2. Bare three-loop contribution to two-point function

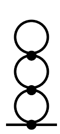
The single component field ($N = 1$) three-loop diagrams for the unrenormalized bare two-loop function are [6]

$$\overline{\Gamma}^{(2)}_{B,3\text{-loop}} = -\frac{1}{4}$$  $$-\frac{1}{12}$$  $$-\frac{1}{4}$$  $$-\frac{1}{8}$$  $$-\frac{1}{8}$$  . (3)

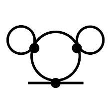
As we are not interested in diagrams proportional to m_B^2, which is the case of tadpole diagram for all orders in the tensor $K_{\mu\nu}$ [20], we see both topologically and mathematically that the last three diagrams have, at least, a tadpole diagram on their expressions as seen below



$$= -\lambda_B^3\int\frac{d^dq_1}{(2\pi)^d}\frac{d^dq_2}{(2\pi)^d}\frac{d^dq_3}{(2\pi)^d}\frac{1}{(q_1^2 + K_{\mu\nu}q_1^{\mu}q_1^{\nu} + m_B^2)^2}\frac{1}{q_2^2 + K_{\mu\nu}q_2^{\mu}q_2^{\nu} + m_B^2} \times \frac{1}{(q_1+q_2+P)^2 + K_{\mu\nu}(q_1+q_2+P)^{\mu}(q_1+q_2+P)^{\nu} + m_B^2}\frac{1}{q_3^2 + K_{\mu\nu}q_3^{\mu}q_3^{\nu} + m_B^2}, \tag{4}$$



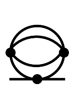
$$= -\lambda_B^3\int\frac{d^dq_1}{(2\pi)^d}\frac{d^dq_2}{(2\pi)^d}\frac{d^dq_3}{(2\pi)^d}\frac{1}{(q_1^2 + K_{\mu\nu}q_1^{\mu}q_1^{\nu} + m_B^2)^2}\frac{1}{(q_2^2 + K_{\mu\nu}q_2^{\mu}q_2^{\nu} + m_B^2)^2}\frac{1}{q_3^2 + K_{\mu\nu}q_3^{\mu}q_3^{\nu} + m_B^2}, \tag{5}$$



$$= -\lambda_B^3\int\frac{d^dq_1}{(2\pi)^d}\frac{d^dq_2}{(2\pi)^d}\frac{d^dq_3}{(2\pi)^d}\frac{1}{(q_1^2 + K_{\mu\nu}q_1^{\mu}q_1^{\nu} + m_B^2)^3}\frac{1}{q_2^2 + K_{\mu\nu}q_2^{\mu}q_2^{\nu} + m_B^2}\frac{1}{q_3^2 + K_{\mu\nu}q_3^{\mu}q_3^{\nu} + m_B^2}. \tag{6}$$

So these diagrams do not contribute to field renormalization.

The second diagram



$$= -\lambda_B^3\int\frac{d^dq_1}{(2\pi)^d}\frac{d^dq_2}{(2\pi)^d}\frac{d^dq_3}{(2\pi)^d}\frac{1}{(q_1^2 + K_{\mu\nu}q_1^{\mu}q_1^{\nu} + m_B^2)^2}\frac{1}{q_2^2 + K_{\mu\nu}q_2^{\mu}q_2^{\nu} + m_B^2}\frac{1}{q_3^2 + K_{\mu\nu}q_3^{\mu}q_3^{\nu} + m_B^2} \times \frac{1}{(q_1+q_2+q_3)^2 + K_{\mu\nu}(q_1+q_2+q_3)^{\mu}(q_1+q_2+q_3)^{\nu} + m_B^2} \tag{7}$$