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Four-dimensional $N = 1$ $F[R]$ supergravity

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ABSTRACT

We propose a supersymmetric generalization of $f[R]$ gravity, calling it $F[R]$ supergravity. We adopt the so-called unimodular supergravity (UMSG). We first give an explicitly invariant Lagrangian $\mathcal{L}_{\rm inv} \equiv \mathcal{L}_{\rm SG} + \mathcal{L}_H$ in dimensions $2 \leqslant {}^\forall D \leqslant 11$, where $\mathcal{L}_H$ is linear in the D-form field strength $H = dC$, while $\mathcal{L}_{\rm SG}$ is the ordinary supergravity Lagrangian. We then establish the total Lagrangian $\mathcal{L}_{\rm tot} \equiv eF[e^{-1}\mathcal{L}_{\rm inv}] + \mathcal{L}_C$, with the constraint term $\mathcal{L}_C$ for the UMSG formulation. As an explicit example, we study $N = 1$ supergravity in four dimensions (4D). We show that the solutions to the field equations for conventional $\mathcal{L}_{\rm SG}$ satisfy the field equation of the new system with $\mathcal{L}_{\rm tot}$. Since the function $F[e^{-1}\mathcal{L}_{\rm inv}]$ is an arbitrary (non)polynomial function of $e^{-1}\mathcal{L}_{\rm inv}$, there can be many other solutions, including those for non-supersymmetric $f[R]$ gravity.

1. Introduction

The higher-derivative modifications of general relativity, such as the so-called $f(R)$ gravity theory, have received much attention, yielding many interesting results in particle physics, cosmology and astrophysics over the last several years [1]. For example, the recent motivation for $f(R)$ theory is to provide an alternative mechanism to explain dark energy [2].

Within the so-called $f(R)$ gravity itself, there have been several distinctive formulations, such as metric, Palatini and metric-affine formalisms. It has been also well known that these so-called $f(R)$ gravity formulations are classically equivalent to Brans–Dicke-type scalar coupled to gravity in the metric formalism [3] and in Palatini formalism [4], as well.

These developments had been limited to non-supersymmetric models. However, since superstring [5] and M-theory [6] are the most likely candidate for the complete description of Nature, it is appropriate to consider the supersymmetrization of the $f(R)$ theory [1]. As a matter of fact, $f(R)$ supergravity has already been presented in four dimensions (4D) [7]. Corresponding to the non-supersymmetric case [1,3,4], the supergravity system [7] is shown to be equivalent to ordinary $N = 1$ Poincaré supergravity coupled to a chiral multiplet *without* the $f(R)$ term, *via* a superfield Legendre–Weyl transformation.

Considering these developments, the next natural question is how to generalize 4D $f(R)$ supergravity, such as to higher dimensions, or extended supersymmetries, etc. Our objective in this Letter is to answer this question. For this purpose, we take an approach completely different from [7] to the supersymmetrization of $f(R)$ theory.

Our formulation is based on a special supergravity Lagrangian $\mathcal{L}_{\rm inv} \equiv \mathcal{L}_{\rm SG} + \mathcal{L}_{\rm H}$, as a sum of the usual supergravity Lagrangian $\mathcal{L}_{\rm SG}$ and an extra term $\mathcal{L}_{\rm H} \equiv (1/D!)\epsilon^{\mu_1\cdots\mu_D} H_{\mu_1\cdots\mu_D}$, made of a D-form field strength H of a $(D-1)$-form potential C. While $\mathcal{L}_{\rm SG}$ is transforming as a surface term, the $\mathcal{L}_{\rm inv}$ is shown to be 'invariant' under local supersymmetry.

It is well known that any Lagrangian of supergravity theory is *not* invariant under local supersymmetry transformation, but it becomes a surface term [8–11]. One of the reasons is that if it *were* invariant under supersymmetry, such a Lagrangian *would* be a constant, since a commutator of two supersymmetries yields a translation operator! This has been the main obstruction against building a Lagrangian which is an arbitrary function of a supergravity Lagrangian. However, we show in this Letter that such a Lagrangian $\mathcal{L}_{\rm inv}$ indeed exists, if we add a special term $\mathcal{L}_H$ made of the maximal-rank field strength H to the conventional supergravity Lagrangian $\mathcal{L}_{\rm SG}$.

Using such an invariant Lagrangian, the total Lagrangian $\mathcal{L}_{\rm tot}$ should be e.g., $eF[e^{-1}\mathcal{L}_{\rm inv}]$, where the function $F[\xi]$ is supposed to be infinitely differentiable by ξ. However, since the determinant of vielbein is generally $e \equiv \det(e_\mu{}^m) \neq 1$, this does not work so easily. The new methodology to overcome this problem is the so-called unimodular supergravity (UMSG) formulation [12]. In this

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formulation, the determinant e is fixed to be unity, and therefore $\delta_Q e = 0$, so that there arises no concern of the contribution to $\delta_Q \mathcal{L}_{tot}$ from $\delta_Q e$ or $\delta_Q e^{-1}$. Eventually, our total Lagrangian is $\mathcal{L}_{tot} \equiv eF[e^{-1}\mathcal{L}_{inv}] + \mathcal{L}_C$, where $\mathcal{L}_C$ is the constraint Lagrangian for the UMSG formulation [12]. As will be shown in the case of $N = 1$ supergravity in 4D, since $\mathcal{L}_{inv} \equiv \mathcal{L}_{SG} + \mathcal{L}_H$ contains the supergravity Lagrangian $\mathcal{L}_{SG}$, which in turn contains the Einstein–Hilbert term: $(-1/4)eR(e)$, our system automatically contains the $f(R)$ in the conventional non-supergravity formulation [1].

UMSG [12] is a supergravity generalization of unimodular gravity (UMG) [13,14] in which the determinant of the vierbein is constrained to be unity. The advantage of UMG is that the cosmological constant is no longer a 'constant', but it is determined as the initial condition.

As an explicit example, we consider the case of $N = 1$ supergravity in 4D. The new term is $\mathcal{L}_H \equiv (1/4!)\epsilon^{\mu\nu\rho\sigma} H_{\mu\nu\rho\sigma}$ added to the conventional $N = 1$ supergravity Lagrangian $\mathcal{L}_{SG}$, where $H = dC$ is the forth-rank field strength of the third-rank potential field C. By fixing the transformation for C, we show that $\delta_Q \mathcal{L}_{inv} = 0$ is indeed realized.

Compared with the past supersymmetric $f(R)$ theory, e.g., [7], our formulation is more general and potentially applicable to many supergravity systems in $2 \leqslant {}^\forall D \leqslant 11$. Even though our formulation relies on the on-shell vanishing feature of $\mathcal{L}_{SG}$ for $N = 1$ supergravity in 4D as a working example, we do not generally need the vanishing of $\mathcal{L}_{SG}$, when applying our mechanism to higher-dimensional system, extended supergravity, or matter-coupled supergravity systems.

2. The total Lagrangian $\mathcal{L}_{tot}$

For the discussion in this section, we consider arbitrary space–time dimensions between $2 \leqslant {}^\forall D \leqslant 11$, where both scalar curvature and Lorentz-covariant supergravity Lagrangians exist. Until the next section, we keep space–time dimensionality D as general as possible. For definiteness, our space–time has the signature $(-, +, +, \dots, +)$.

As has been stated, any Lagrangian, say $\mathcal{L}_{SG}$, in supergravity theory in $2 \leqslant {}^\forall D \leqslant 11$ is *not* invariant under local supersymmetry, but it transforms to a total divergence. However, if it is always only a total divergence, we can add a certain term to $\mathcal{L}_{SG}$, such that the total Lagrangian $\mathcal{L}_{inv}$ becomes invariant: $\delta_Q \mathcal{L}_{inv} = 0$, at least by the use of its own field equations.

The question is how to fix such a term. Most naturally, it should be a total divergence again, so that the most natural choice is the maximal-rank tensor in a given space–time dimension D. Consider a $(D-1)$-form potential field $C_{\mu_1\cdots\mu_{D-1}}$ whose field strength is

$$H_{\mu_1\cdots\mu_D} \equiv D\partial_{[\mu_1} C_{\mu_2\cdots\mu_D]}, \tag{2.1}$$

and add the Lagrangian $\mathcal{L}_H$ to $\mathcal{L}_{SG}$, to get an 'invariant' Lagrangian $\mathcal{L}_{inv}$:

$$\mathcal{L}_H \equiv +\frac{1}{D!}\epsilon^{\mu_1\cdots\mu_D} H_{\mu_1\cdots\mu_D}, \qquad \mathcal{L}_{inv} \equiv \mathcal{L}_{SG} + \mathcal{L}_H. \tag{2.2}$$

The supersymmetry transformation rule for C is determined in such a way that $\delta_Q \mathcal{L}_{inv} = 0$. Since $\delta_Q \mathcal{L}_{SG}$ is supposed to be a total divergence, we can introduce the new field ζ^μ as

$$\begin{aligned}\delta_Q \mathcal{L}_{SG} &= \partial_\mu\big[e(\bar\epsilon\zeta^\mu)\big] \\ &= \frac{1}{(D-1)!}\epsilon^{\mu\nu_1\cdots\nu_{D-1}}\partial_\mu(\bar\epsilon\eta_{\nu_1\cdots\nu_{D-1}}),\end{aligned} \tag{2.3}$$

where η and ζ are Hodge dual to each other:

$$\eta_{\mu_1\cdots\mu_{D-1}} \equiv (-1)^D \epsilon_{\mu_1\cdots\mu_{D-1}\nu} e^{-1}\zeta^\nu. \tag{2.4}$$

Then the transformation rule for C-field is determined to be

$$\delta_Q C_{\mu_1\cdots\mu_{D-1}} = -(\bar\epsilon\eta_{\mu_1\cdots\mu_{D-1}}). \tag{2.5}$$

It is now easy to confirm the exact invariance $\delta_Q \mathcal{L}_{inv} = 0$, where *no* surface term is ignored:

$$\begin{aligned}0 &\overset{?}{=} \delta_Q \mathcal{L}_{inv} = \delta_Q\left(\mathcal{L}_{SG} + \frac{1}{D!}\epsilon^{\mu\nu_1\cdots\mu_D} H_{\mu_1\cdots\mu_D}\right) \\ &= \partial_\mu\big[e(\epsilon\zeta^\mu)\big] + \frac{1}{D!}\epsilon^{\mu\nu_1\cdots\nu_{D-1}} D\partial_\mu(\delta_Q C_{\nu_1\cdots\nu_{D-1}}) \\ &= \frac{1}{(D-1)!}\epsilon^{\mu\nu_1\cdots\nu_{D-1}}\partial_\mu(\bar\epsilon\eta_{\nu_1\cdots\nu_{D-1}}) \\ &\quad + \frac{1}{(D-1)!}\epsilon^{\mu\nu_1\cdots\nu_{D-1}}\partial_\mu(\delta_Q C_{\nu_1\cdots\nu_{D-1}}) \\ &= \frac{1}{(D-1)!}\epsilon^{\mu\nu_1\cdots\nu_{D-1}}\partial_\mu(\bar\epsilon\eta_{\nu_1\cdots\nu_{D-1}}) \\ &\quad + \frac{1}{(D-1)!}\epsilon^{\mu\nu_1\cdots\nu_{D-1}}\partial_\mu\big[-(\bar\epsilon\eta_{\nu_1\cdots\nu_{D-1}})\big] \\ &= 0 \quad (Q.E.D.). \qquad \square\end{aligned} \tag{2.6}$$

Once an invariant Lagrangian $\mathcal{L}_{inv}$ has been established, it is straightforward to generalize it to an arbitrary (non)polynomial function. However, there is one obstruction. Namely, the Lagrangian $\mathcal{L}_{inv}$ transforms as a scalar density instead of a scalar under general coordinate transformation. Therefore, we are forced to consider the structure $F[e^{-1}\mathcal{L}_{inv}]$, because the argument of $F[\xi]$ should be a scalar instead of scalar density, so that the total expression $F[e^{-1}\mathcal{L}_{inv}]$ is again a scalar. Therefore the appropriate total Lagrangian to consider is $\mathcal{L}_{tot} \equiv eF[e^{-1}\mathcal{L}_{inv}]$ carrying e for a scalar density for $\mathcal{L}_{tot}$.

However, now the price to be paid is that the factor e^{-1} also contribute to the variation under supersymmetry, because $\delta_Q e \neq 0$. To solve this problem, we need to rely on the so-called UMSG formulation [12], in which the determinant of the vielbein $e_\mu{}^m$ is constrained to unity: $e \equiv \det(e_\mu{}^m) = 1$, and therefore $\delta_Q e = 0$, so that no undesirable contribution arises to $\delta_Q \mathcal{L}_{tot}$.

Historically, the original motivation of UMG [13,14] was its advantage for the cosmological constant problem, because in UMG the cosmological constant is given as an initial condition, instead of a fine-tuning in conventional general relativity. UMG yields the ordinary Einstein gravitational field equation with a cosmological constant [13,14], so that there is *no* problem with passing the standard tests of general relativity, as long as the cosmological constant is desirably small. In this sense, UMG [13,14] has more advantage than conventional general relativity.

Following the original UMSG formulation [12], our total Lagrangian $\mathcal{L}_{tot}$ is now composed of two terms, with the constraint Lagrangian $\mathcal{L}_C$:

$$\mathcal{L}_{tot} \equiv eF\big[e^{-1}\mathcal{L}_{inv}\big] + \mathcal{L}_C, \tag{2.7}$$

$$\mathcal{L}_C \equiv \Lambda(e-1) + e(\bar\rho\gamma^\mu\psi_\mu). \tag{2.8}$$

The fields Λ and ρ are Lagrange-multiplier fields [14,12].

The vielbein and the gravitino are subject to universal transformation rule for supergravity in $2 \leqslant {}^\forall D \leqslant 11$[1]:

[1] Generally, there can be more fields other than $e_\mu{}^m$ and ψ_μ. Even if they exist, they do not seem to interfere our argument below. The non-interference of these fields is similar to the original UMSG [12].