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A novel subtraction scheme for double-real radiation at NNLO ☆

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A B S T R A C T

A general subtraction scheme, STRIPPER (SecToR Improved Phase sPacE for real Radiation), is derived for the evaluation of next-to-next-to-leading order (NNLO) QCD contributions from double-real radiation to processes with at least two particles in the final state at leading order. The result is a Laurent expansion in the parameter of dimensional regularization, the coefficients of which should be evaluated by numerical Monte Carlo integration. The two main ideas are a two-level decomposition of the phase space, the second one factorizing the singular limits of amplitudes, and a suitable parameterization of the kinematics allowing for derivation of subtraction and integrated subtraction terms from eikonal factors and splitting functions without non-trivial analytic integration.

1. Introduction

Compared to the number of phenomenological applications, where NNLO QCD corrections are indispensable, the effort put into the construction of general subtraction schemes for real radiation at this level of perturbation theory is astounding. The main problems encountered are either, that the method is not general and requires tedious adaptation to every specific problem, or that there are many highly non-trivial divergent dimensionally regulated integrals to evaluate.

This state of affairs is to be contrasted with the comforting situation at the NLO level, where general solutions have been available for long. The approach of choice is that of Catani and Seymour [1], later extended to massive states [2] and arbitrary polarizations in real radiation [3]. In fact, we would encourage the non-expert reader to consult the original paper [1], since it gives a thorough discussion of all aspects of the problem. The present Letter assumes such knowledge.

The main features of the Catani–Seymour subtraction scheme are the smooth interpolation of the subtraction terms between soft and collinear limits and independence from the phase space parameterization achieved by a remapping of phase space points onto the reduced phase space with one parton less. There is another scheme at NLO derived by Kunszt, Frixione and Signer (FKS) [4], which is vastly different on the conceptual side. Here, the phase space is first decomposed into sectors (originally with the help of the jet function; an independent decomposition has been proposed in [5]), and then parameterized with energy and angle variables for easy extraction of the subtraction terms. Precisely these ideas will turn out to be crucial for the scheme that we shall derive.

Many approaches have been proposed at NNLO. The most successful ones are Sector Decomposition [6–8] and Antenna Subtraction [9]. Sector decomposition is conceptually entirely different from the NLO methods cited above. The idea is to derive a Laurent expansion for the given amplitude, by first ingeniously parameterizing the complete phase space, mapping it onto the unit hypercube, and then dividing it into simplexes, in which singularities are factorized. The parameterization is adapted to different singularity structures for different diagram classes. A detailed description on the particular example of Higgs boson production can be found in [10]. The main drawback is that one has to repeat everything for a new problem, which is relatively easy, if it involves the same number of particles in the final state with the same mass distribution, but is expected to be a major effort otherwise. Antenna subtraction, on the other hand, uses complete matrix elements of simpler processes as building blocks for the subtraction. In this way, the integration over the unresolved particle phase space can be made largely with multi-loop methods. It is this latter part that involves most work, but the result is general and can be applied to other processes without modification. The drawback of this approach is the efficiency loss due to the fact that azimuthal correlations characteristic of collinear limits are not taken into account, as the simplified matrix elements correspond to unpolarized

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scattering. There are also other methods, which are either specific to a class of problems, such as that of [11], which solves the problem for the production of colorless states, or still require the integration of the subtraction terms, as for example in [12] and [13].

The purpose of this Letter is to present a new approach, which should provide a method both general and simple to derive. To this end, we need to specify, which problems are essentially difficult and which are not. Unlike at NLO, there are in fact two different problems involving real radiation at NNLO. One involves the emission of one additional parton (in comparison to leading order) out of virtual diagrams. Usually called single-real radiation, since only one parton can become unresolved, it can be treated by any of the NLO methods. Of course, the subtraction terms require a slight modification. In fact, we believe that the FKS approach will provide the result with the least effort. By this argument, we shall ignore single-real radiation and concentrate on double-real radiation. The latter problem involves two unresolved partons and only tree-level matrix elements. The method presented will not depend on the nature of the initial state. Let us, however, consider the more difficult case of hadronic collisions. Due to the factorization theorem, the result is a convolution of partonic cross sections with Parton Distribution Functions (PDFs). As long as the partonic cross section is an ordinary function, this convolution can be considered independently (see Section 2.1). Indeed, in actual Monte Carlo generators, one first generates the fractions of hadron momenta to be assigned to the partons and then works in the center-of-mass frame of the partons, multiplying every event by the PDF weight only at the end. This is the point of view that we shall adopt as well. To summarize: we will consider the derivation of the Laurent expansion of the double-real radiation contributions for fixed initial parton energies.

The main concept of our approach is to mix some ideas of the FKS NLO subtraction scheme with those of Sector Decomposition. We will decompose the phase space in two stages. At the first stage, we will divide the problem into triple- and double-collinear sectors. Then, using an energy-angle parameterization of the phase space, we will perform an ordinary sector decomposition mimicking the physical singular limits. The crucial novelty is that we will show how to obtain general subtraction terms from the last sector decomposition. Here, we will use the knowledge of NNLO singular behavior of QCD amplitudes as studied in [14–16] and summarized in [17]. While we will not give explicit expressions for the subtraction terms, a task impossible in a Letter due to the multitude of cases, it is easy to rederive them following the description.

In the next section, we will derive the scheme on the example of massive particle production. The reason for this restriction on the final state is that this Letter will be followed by a companion publication with process specific information and numerical results for our first application: top quark pair production. A subsequent section will, however, present the generalization to arbitrary final states.

2. Massive final states at leading order

2.1. Phase space

Let us assume that there are only two massless partons in the final state, the other final state particles being massive. In case there is only one massive final state, the presence of soft singularities leads to a cross section, which is a distribution in the partonic center-of-mass energy squared, s. We will not take this possibility into account, since it has already been extensively studied for all processes of phenomenological interest and is a special case involving additional complications. The cross section will, therefore, be an ordinary function of s. The considered process corresponds to the following kinematical configuration

$$p_1 + p_2 \to k_1 + k_2 + q_1 + \cdots + q_n, \tag{1}$$

with

$$s = (p_1 + p_2)^2, \qquad p_1^2 = p_2^2 = k_1^2 = k_2^2 = 0, \qquad q_i^2 = m_i^2 \neq 0, \quad i = 1, \ldots, n,\ n \geqslant 2, \tag{2}$$

where $p_1, p_2, k_1, k_2, q_1, \ldots, q_n$ are d-dimensional momentum vectors. The d-dimensional phase space can be written as

$$\int d\Phi_{n+2} = \int \frac{d^{d-1}k_1}{(2\pi)^{d-1}2k_1^0} \frac{d^{d-1}k_2}{(2\pi)^{d-1}2k_2^0} \prod_{i=1}^{n} \frac{d^{d-1}q_i}{(2\pi)^{d-1}2q_i^0} (2\pi)^d \delta^{(d)}(k_1 + k_2 + q_1 + \cdots + q_n - p_1 - p_2). \tag{3}$$

The above definition suggests a factorization of the phase space for this problem into the three-particle production phase space of the two massless partons together with an object with invariant mass Q^2, and a decay phase space of the composite with momentum Q into the massive particles. Such a factorization is motivated by the fact that most divergences are due to the vanishing of invariants involving only massless states momenta. The divergences not belonging to this class are soft and involve the massive states momenta. In this case, the inverse propagators responsible for the singularities vanish proportionally to a linear combination of the energy components of k_1 and k_2. This will force us to use these energy components as part of the phase space parameterization. At this point the phase space can be written as

$$\int d\Phi_{n+2} = \int \frac{dQ^2}{2\pi} \int \frac{d^{d-1}k_1}{(2\pi)^{d-1}2k_1^0} \frac{d^{d-1}k_2}{(2\pi)^{d-1}2k_2^0} \frac{d^{d-1}Q}{(2\pi)^{d-1}2Q^0} (2\pi)^d \delta^{(d)}(k_1 + k_2 + Q - p_1 - p_2)$$
$$\times \int \prod_{i=1}^{n} \frac{d^{d-1}q_i}{(2\pi)^{d-1}2q_i^0} (2\pi)^d \delta^{(d)}(q_1 + \cdots + q_n - Q). \tag{4}$$

The integrations over Q can be performed by exploiting the δ-function, which leaves

$$\int d\Phi_{n+2} = \int \frac{d^{d-1}k_1}{(2\pi)^{d-1}2k_1^0} \frac{d^{d-1}k_2}{(2\pi)^{d-1}2k_2^0} \int \prod_{i=1}^{n} \frac{d^{d-1}q_i}{(2\pi)^{d-1}2q_i^0} (2\pi)^d \delta^{(d)}(q_1 + \cdots + q_n - Q) \equiv \int d\Phi_3 \int d\Phi_n(Q), \tag{5}$$

where