Physics Letters B 693 (2010) 330–333

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Towards the MSSM from F-theory

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ARTICLE INFO

Article history:
Received 25 May 2010
Received in revised form 16 August 2010
Accepted 17 August 2010
Available online 21 August 2010
Editor: M. Cvetič

Keywords:
MSSM
F-theory
Compactification
Three generations
R-parity

ABSTRACT

We study the MSSM in F-theory. Its group is the commutant to a structure group $SU(5) \times U(1)_Y$ of a gauge bundle in E_8. The spectrum contains three generations of quarks and leptons plus vectorlike electroweak and colored Higgses. The minimal MSSM Yukawa couplings with matter parity is obtained at the renormalizable level.

1. Introduction

The purpose of this Letter is to describe the Minimal Supersymmetric Standard Model (MSSM) in F-theory. String theory provides self-consistent ultraviolet completion of the gauge theory. For example, the constraint of anomaly freedom in the low energy theory originates from the finiteness of string one-loop amplitude.

F-theory is defined by identifying S-duality of IIB theory with the symmetry of a torus, lifting the gauge symmetry to geometry [1]. To have four-dimensional theory with $\mathcal{N} = 1$ supersymmetry, we compactify F-theory on Calabi–Yau fourfold that is elliptic fibered. Gauge bosons are localized on a complex surface S, along which the fiber is singular. The structure of the singularities has correspondence to that of the corresponding group, so that the symmetry breaking and enhancement are described by geometric transition [2]. Analogous to the bifundamental representations at D-brane intersections, matter fermions come from the 'off-diagonal' components under branching of the gaugino of E_8 [3], localized along matter curves [4].

So far there have been constructions of Grand Unified Theories (GUTs) mainly based on the simple group $SU(5)$, and further attention has been paid on the flavor sector [5–7]. Compared to field theoretic GUT, presumably its best merit is that we do not need adjoint Higgses for breaking down to SM. Instead, we can turn on a flux in the hypercharge $U(1)_Y$ direction, evading complicated GUT vacuum configuration. This flux does not break hypercharge interaction, if a certain topological condition is satisfied [6]. However, further breaking down to SM can potentially lead to chiral multiplet containing X-boson, the off-diagonal component of the adjoint of $SU(5)$, by the above mechanism localizing light matters.

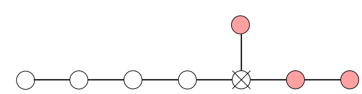

Fig. 1. The Standard Model group (filled) is obtained as the commutant to $SU(5)_\perp \times U(1)_Y$ background in E_8.

Another obvious approach that we take here is to start with the group $SU(3) \times SU(2) \times U(1)_Y$. The SM group lies along the series of exceptional groups of E_n-type, and we take $E_3 \times U(1)_Y$ group inside E_8, shown in Fig. 1, guaranteeing the correct field contents and quantum numbers. Such E_n-series is naturally predicted by F-theory and heterotic string. For this we need a description on semisimple groups [8,9]. If $U(1)_Y$ is constructed via geometry and there is no flux along this part, embeddability to $SU(5)$ GUT singularity guarantees anomaly free spectrum and we do not worry about its breaking by Green–Schwarz mechanism.

2. The Standard Model surface

A gauge group is described by a singular fiber sharing the same name, which is read off from Tate's table [10]. We claim that the singularity describing the SM group is

$$y^2 = x^3 + (b_5 + b_4 a_1)xy + (b_3 + b_2 a_1)(a_1 b_5 + z)zy$$

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0370-2693/$ – see front matter
doi:10.1016/j.physletb.2010.08.042

$$+ (b_4 + b_3 a_1)x^2 z + (b_2 - b_0 a_1^2)(a_1 b_5 + z)xz^2 + b_0(a_1 b_5 + z)^2 z^3. \quad (1)$$

For the total space to be Calabi–Yau, the equation should satisfy topological conditions: z, a_1, b_n are respectively sections of $\mathcal{O}(S), \mathcal{O}(K_B + S), \mathcal{O}((n-6)K_B + (n-5)S)$ in the base B of elliptic fibration, and K_B is its canonical bundle. The surface S is located at $z = 0$, since the vanishing discriminant of (1)

$$\Delta = (b_5 + a_1 b_4)^3 a_1^2 b_5^2 P_{d_\circ^c} P_{u_\circ^c} z^3 + a_1 b_5 Q z^4 + O(z^5) \quad (2)$$

signals a singular fiber. We need globally defined sections in B, since the simple group components would be laid away from S.

Eq. (1) is a deformation of E_8 singularity. Surveying the degrees of the coefficients in (1), Tate's table shows it is generically an $SU(3)$ singularity. However the parameters are specially tuned, so the actual symmetry is larger [8]. Vanishing of each coefficient factor triggers gauge symmetry enhancement, implying matter localization [2]. We see shortly that the parameters $a_1 \equiv P_X$, $b_5 \equiv P_{q_\circ}$, $P_{d_\circ^c}$ and $P_{u_\circ^c}$ are respectively related to X-boson and subscripted quarks, which are all the charged fields under $SU(3)$. We can see, neither Q nor $O(z^5)$ is proportional to a_1 or b_5, hence $P_{d_\circ^c} P_{u_\circ^c}$ and $a_1 b_5 \to 0$ respectively enhance the symmetry to $SU(4)$ and $SU(5)$.

To see it contains also $SU(2)$, we change the variable $z' = z + a_1 b_5$, in which (2) becomes

$$\Delta = \left((b_5 - a_1 b_4)^2 - 4a_5^2 b_3 b_5\right)^2 a_1^3 b_5^3 P_{l_\circ} z'^2 + a_1^2 b_5^2 P' z'^3 + a_1 b_5 Q' z'^4 + O(z'^5). \quad (3)$$

Here $P_{l_\circ}$ is related to the lepton doublet. Since neither of P', Q' nor $O(z'^5)$ is proportional to a_1 or b_5, vanishing $P_{l_\circ}$ and $a_1 b_5$ respectively enhances the gauge symmetry respectively to $SU(3)$ and $SU(5)$. The position $z' = 0$ is off from the position of S, as a 'backreaction' under the symmetry breaking from $SU(5)$, but it is still linearly equivalent to S. We can see, to make the embracing $SU(5)$ truly traceless, we need also a backreaction $z \to z - 2a_1 b_5/5$ to make the center of mass of the total brane lie on S.

The factorization structure of $P_{u_\circ^c} P_{d_\circ^c}$ and $P_X P_{q_\circ} = a_1 b_5$ hints the existence of an additional $U(1)_Y$, otherwise we have only single factors for colored singlet and colored doublet, respectively. However to guarantee the existence of such $U(1)_Y$ we should check the global factorization structure of (1) [11]. Above all, the most general deformation is by $a_1 \to 0$, implying the embedding of our symmetry to a general $SU(5)$ GUT singularity in the literature [3]. Being its deformation, the factorization structure of $SU(3) \times SU(2) \times U(1)_Y$ is stably preserved against higher order perturbation in $O(z^6)$. Lying along the E_n unification series $E_3 \times U(1)_Y \to E_4 \to E_8$, this SM group is the only possibility.

3. Matter contents

The broken symmetry is $SU(5)_\perp \times U(1)_Y$ 'structure group' whose commutant in E_8 is the Standard Model group [12]. The spectral cover geometrically describes it (or the dual to gauge bundle) satisfying supersymmetry conditions [13]. Roughly, it generalizes the notion of branes whose symmetry broken by recombination, so sometimes called by flavor brane stack, intersecting S along the matter curves. It is described by factorized spectral cover $C = C_X \cup C_5$:

$$(a_0 s + a_1)\left(b_0 s^5 - a_0^{-1} a_1 b_0 s^4 + b_2 s^3 + b_3 s^2 + b_4 s + b_5\right) = 0, \quad (4)$$

where s transforms as $-c_1 \equiv K_S$, the canonical bundle of S. We also define $-t$ as the normal bundle to S in B. (4) is embedded in a compact threefold,[1] whose projection to S we denote π. The parameters are the ones used in (1) projected on S: Using adjunction formula b_m are sections of $(6-m)c_1 \equiv \eta - mc_1$ and the combination $a_1 b_0 / a_0$ is a section of $5c_1 - t$, with which the full equation for C has no s^5 term showing the embeddability to a $SU(6)_\perp$ structure group.[2] Using this embedding structure, one can be convinced that the parameters in (1) are the only possible combinations. We can recover the conventional $SU(5)$ GUT by $a_1 \to 0$ in (2), making its structure group traceless, reducing the above C_5 exactly to the standard one used in $SU(5)$ GUT. The fields charged under two groups satisfy the Green–Schwarz relations [21,8], fixing $a_0 = 1$ to be trivial section.

We get the matter spectrum from the decomposition of the adjoint **248** of E_8

$$(\mathbf{8},\mathbf{1},\mathbf{1}) + (\mathbf{1},\mathbf{3},\mathbf{1}) + (\mathbf{1},\mathbf{1},\mathbf{1}) + (\mathbf{1},\mathbf{1},\mathbf{24}) + X(\mathbf{3},\mathbf{2},\mathbf{1})_{-5/6} + q_\circ(\mathbf{3},\mathbf{2},\mathbf{5})_{1/6} + d_\circ^c(\bar{\mathbf{3}},\mathbf{1},\mathbf{10})_{1/3} + u_\circ^c(\bar{\mathbf{3}},\mathbf{1},\mathbf{5})_{-2/3} + l_\circ(\mathbf{1},\mathbf{2},\mathbf{10})_{-1/2} + e_\circ^c(\mathbf{1},\mathbf{1},\bar{\mathbf{5}})_{-1},$$

up to Hermitian conjugates. Matter fields are obtained as 'off-diagonal' components of the branching [2].

They are further identified by local gauge symmetry enhancement directions. For this, we parameterize their localizing curves using parameters $t_1, t_2, \dots, t_5$ having one-to-one correspondence with the five weights of **5** of $SU(5)_\perp$ and t_6 with $U(1)_Y$. For example, the X-boson appears from a local symmetry enhancement to $SU(5)$, controlled by $t_6 \to 0$, agreeing with the above. However the physical parameters are only the coefficients in the spectral cover (4): b_n/b_0 are the elementary symmetric polynomials of degree k, of t_1, t_2, t_3, t_4, t_5, and $a_1 \equiv P_X \sim t_6$ [5,3]. $P_{q_\circ}$ is localized along the curve $b_5/b_0 \sim t_1 t_2 t_3 t_4 t_5 = 0$ and $P_{u_\circ^c} \sim \prod_{i=1}^5 (t_i + t_6)$, $P_{d_\circ^c} \sim \prod_{i<j}^5 (t_i + t_j)$, $P_{l_\circ} \sim \prod_{i<j}^5 (t_i + t_j + t_6)$. The counting of **10** and $\overline{\mathbf{10}}$ agrees thanks to the relation

$$t_i + t_j + t_6 = -t_k - t_l - t_m, \quad \epsilon_{ijklm} \neq 0. \quad (5)$$

It is easy to see that the parameters $P_{m_\circ}, m = X, q, u^c, d^c, l$, here calculated from group theory, agree with those in (2) and (3). Again $t_6 \to 0$ reduces the matter curves to those of $SU(5)$, so the $SU(5)$ GUT structure is preserved.

In the perturbative picture, parallel D-branes do not intersect. Here, the base of elliptic fibration has a similar structure generalizing Hirzeburch surface, where the zero section of the fiber has nonzero 'self-intersection'. Thus $SU(3)$ and $SU(2)$ components intersect yielding chiral fermions X and q. In the sense of regarding the intersecting branes as connected cycles [15], this is not so different.

By dimensional reduction, we obtain Yukawa coupling from covariant derivatives of gaugino of an enhanced group, as usual [5]. So the gauge invariance guarantees the presence of Yukawa couplings. They are

$$\begin{aligned}
&l h_d e^c: \quad (t_i + t_j + t_6) + (t_k + t_l + t_6) + (t_m - t_6) = 0,\\
&q h_u u^c: \quad (t_i) + (-t_i - t_j - t_6) + (t_j + t_6) = 0,\\
&q h_d d^c: \quad \begin{cases} (t_m) + (t_k + t_l + t_6) + (t_i + t_j) = 0, \\ (t_m) + (-t_m - t_i - t_j) + (t_i + t_j) = 0, \end{cases}
\end{aligned} \quad (6)$$

where all the indices run from 1 to 5, and all different.

[1] The compact (non-Calabi–Yau) threefold, a projectivized fiber $\pi : \mathbb{P}(\mathcal{O}_S \oplus K_S) \to S$, with the trivial bundle $\mathcal{O}_S$ and the canonical bundle K_S.

[2] Since the trace part of $SU(5)_\perp$ is the $U(1)_Y$ degree of freedom, it is sometimes called $S[U(5) \times U(1)]$.