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Ordering of spin-$\frac{1}{2}$ excitations of the nucleon in lattice QCD

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ABSTRACT

We present results for the negative parity low-lying state of the nucleon, $N\frac{1}{2}^-$ (1535 MeV) S_{11}, from a variational analysis method. The analysis is performed in quenched QCD with the FLIC fermion action. The principal focus of this Letter is to explore the level ordering between the Roper (P_{11}) and the negative parity ground (S_{11}) states of the nucleon. Evidence of the physical level ordering is observed at light quark masses. A wide variety of smeared-smeared correlation functions are used to construct correlation matrices. A comprehensive correlation matrix analysis is performed to ensure an accurate isolation of the $N\frac{1}{2}^-$ state.

1. Introduction

Lattice QCD is very successful in computing many properties of hadrons from first principles. In particular, the ground state of the hadron spectrum is a well understood problem [1]. However, the excited states still prove a significant challenge. One of the long-standing puzzles in hadron spectroscopy has been the low mass of the first positive parity excitation of the nucleon, known as the Roper resonance, $N\frac{1}{2}^+$ (1440 MeV)P_{11}, compared to the lowest-lying negative parity partner, $N\frac{1}{2}^-$ (1535 MeV)S_{11}. This phenomenon cannot be observed in constituent or valence quark models where the lowest-lying odd parity state naturally occurs below the $N=\frac{1}{2}^+$ state. Similar difficulties in the level orderings also appear for the $J^P=\frac{3}{2}^+\Delta^*(1600)$ and $\frac{1}{2}^+\Sigma^*(1690)$ resonances.

There has been extensive research focussing on the issue of the level ordering problem using the lattice QCD approach [2–14]. One of the state-of-the-art approaches that has been used extensively in hadron spectroscopy is the 'variational method' [15,16], which is based on a correlation matrix analysis. In the past the isolation of the Roper resonance was elusive with this method. However, in Refs. [17,18] a low-lying Roper state has been identified using a correlation matrix construction with smeared-smeared correlators. Our work there motivates us to investigate the long-standing level ordering problem using the same techniques on the same lattice.

In contrast to the positive parity ground state of the nucleon, $N\frac{1}{2}^+$, which has a large plateau over Euclidean-time, the correlation functions for the negative parity ground state, $N\frac{1}{2}^-$, are short-lived giving shorter plateaus at earlier Euclidean times. Therefore, the standard analysis to extract the $N\frac{1}{2}^-$ ground state from small Euclidean-times may provide a mixture of ground and excited states. On the other hand, the variational method accounts for the presence of excited states in the correlation functions via correlation matrices. The masses of the energy states are then obtained by projecting the correlation matrix to eigenstates [17] providing a robust approach for extracting the energy states. In addition, considering several bases in constructing different correlation matrices provides a substantial verification of the analysis technique, in allowing the consistency of the energy states over the different basis and the reliability of the extracted eigenstates energies to be explored.

In this Letter, we use the same approach as that of Ref. [17] to isolate the negative parity states of the nucleon. In particular, we focus on the negative parity state to explore the level ordering problem.

Various sweeps of gauge invariant Gaussian smearing [19] are used to construct a smeared-smeared correlation function basis to form correlation matrices.

This Letter is arranged as follows: Section 2 contains a brief description of the variational method. Lattice details are in Section 3. Results are discussed in Section 4 and conclusions are presented in Section 5.

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2. Variational method

The two point correlation function matrix for $\vec{p}=0$ can be written as

$$G_{ij}^{\pm}(t) = \sum_{\vec{x}} \mathrm{Tr}_{\mathrm{sp}}\left\{\Gamma_{\pm}\langle\Omega|\chi_i(x)\bar{\chi}_j(0)|\Omega\rangle\right\}, \tag{1}$$

$$= \sum_{\alpha} \lambda_i^{\alpha}\bar{\lambda}_j^{\alpha} e^{-m_{\alpha}t}, \tag{2}$$

where, Dirac indices are implicit. Here, λ_i^{α} and $\bar{\lambda}_j^{\alpha}$ are the couplings of interpolators χ_i and $\bar{\chi}_j$ at the sink and source respectively and α enumerates the energy eigenstates with mass m_{α}. $\Gamma_{\pm} = \frac{1}{2}(\gamma_0 \pm 1)$ projects the parity of the eigenstates.

Since the only t dependence comes from the exponential term, one can seek a linear superposition of interpolators, $\bar{\chi}_j u_j^{\alpha}$, such that,

$$G_{ij}(t_0+\Delta t)\, u_j^{\alpha} = e^{-m_{\alpha}\Delta t}\, G_{ij}(t_0)\, u_j^{\alpha}, \tag{3}$$

for sufficiently large t_0 and $t_0+\Delta t$. More detail can be found in Refs. [6,20,21]. Multiplying the above equation by $[G_{ij}(t_0)]^{-1}$ from the left leads to an eigenvalue equation,

$$\left[\left(G(t_0)\right)^{-1} G(t_0+\Delta t)\right]_{ij} u_j^{\alpha} = c^{\alpha}\, u_i^{\alpha}, \tag{4}$$

where $c^{\alpha} = e^{-m_{\alpha}\Delta t}$ is the eigenvalue. Similar to Eq. (4), one can also solve the left eigenvalue equation to recover the v^{α} eigenvector,

$$v_i^{\alpha}\left[G(t_0+\Delta t)\left(G(t_0)\right)^{-1}\right]_{ij} = c^{\alpha} v_j^{\alpha}. \tag{5}$$

The vectors u_j^{α} and v_i^{α} diagonalize the correlation matrix at time t_0 and $t_0+\Delta t$ making the projected correlation matrix,

$$v_i^{\alpha} G_{ij}^{\pm}(t) u_j^{\beta} \propto \delta^{\alpha\beta}. \tag{6}$$

The parity projected, eigenstate projected correlator,

$$G_{\pm}^{\alpha} \equiv v_i^{\alpha} G_{ij}^{\pm}(t) u_j^{\alpha}, \tag{7}$$

is then analyzed using standard techniques to obtain the masses of different states.

3. Simulation details

Our lattice ensemble is the same as that explored in Ref. [17]. It consists of 200 quenched configurations with a lattice volume of $16^3 \times 32$. Gauge field configurations are generated by using the DBW2 gauge action [22,23] and an $\mathcal{O}(a)$-improved FLIC fermion action [24] is used to generate quark propagators. This action has excellent scaling properties and provides near continuum results at finite lattice spacing [25]. The lattice spacing is $a = 0.127$ fm, as determined by the static quark potential, with the scale set using the Sommer scale, $r_0 = 0.49$ fm [26]. In the irrelevant operators of the fermion action we apply four sweeps of stout-link smearing to the gauge links to reduce the coupling with the high frequency modes of the theory [27] providing $\mathcal{O}(a)$ improvement [25]. We use the same method as in Refs. [28,20] to determine fixed boundary effects, and the effects are significant only after time slice 25 in the present analysis. Various sweeps of gauge invariant Gaussian smearing [19] (1, 3, 7, 12, 16, 26, 35, 48 sweeps) corresponding to rms radii, in lattice units, of 0.6897, 1.0459, 1.5831, 2.0639, 2.3792, 3.0284, 3.5237, 4.1868, are applied at the source ($t = 4$) and at the sink. This is to ensure a large range of overlaps of the interpolators with the lower-lying states. The analysis is performed on eight different quark masses corresponding to pion masses of $m_{\pi} = \{0.797, 0.729, 0.641, 0.541, 0.430, 0.380, 0.327, 0.295\}$ GeV. The error analysis is performed using the jackknife method, with the χ^2/dof obtained via a covariance matrix analysis method. Our fitting method is discussed extensively in Ref. [20].

The nucleon interpolators we consider are,

$$\chi_1(x) = \epsilon^{abc}\left(u^{Ta}(x) C\gamma_5 d^b(x)\right) u^c(x), \tag{8}$$

$$\chi_2(x) = \epsilon^{abc}\left(u^{Ta}(x) C d^b(x)\right)\gamma_5 u^c(x). \tag{9}$$

We use the Dirac representation of the gamma matrices in our analysis.

4. Results

We consider several 3×3, 4×4, 6×6 and 8×8 correlation matrices. Each matrix is constructed with different sets of correlation functions, each set element corresponding to a different numbers of sweeps of gauge invariant Gaussian smearing at the source and sink of the $\chi_1\bar{\chi}_1$, $\chi_2\bar{\chi}_2$ and $\chi_1\chi_2$ correlators [18]. This provides a large basis of operators with a variety of overlaps among energy states.

We consider five smearing combinations (bases) $\{1 = (7, 16, 26), 2 = (7, 16, 35), 3 = (12, 16, 26), 4 = (12, 26, 35), 5 = (16, 26, 35)\}$ for 3×3 correlation function matrices and four combinations $\{1 = (1, 12, 26, 48), 2 = (3, 12, 26, 35), 3 = (3, 12, 26, 48), 4 = (12, 16, 26, 35)\}$ for 4×4 matrices, of $\chi_1\bar{\chi}_1$ correlation functions. In the latter case these four combinations are found optimal for the reliable extraction of the low-lying energy states shown in Ref. [17]. Including the χ_2 interpolator, which vanishes in the non-relativistic limit [29,30], in correlation matrix analysis provides extra challenges. Nonetheless, we consider this interpolator for the reliable extraction of the negative parity ground state mass. The same bases, as discussed above for the $\chi_1\bar{\chi}_1$ analysis, are also considered for the 3×3 and 4×4 correlation matrices of $\chi_2\bar{\chi}_2$ correlation functions. We also consider four smearing combinations $\{1 = (3, 12, 26), 2 = (3, 16, 48), 3 = (7, 16, 35), 4 = (12, 16, 26)\}$ of 6×6 and four combinations $\{1 = (3, 12, 26, 48), 2 = (7, 12, 26, 35), 3 = (7, 16, 26, 35), 4 = (7, 16, 35, 48)\}$ for 8×8 matrices of $\chi_1\chi_2$ correlation functions.

In Fig. 1, masses from the projected correlation functions and from the eigenvalues are shown for the 3×3 and 4×4 correlation matrices. We refer to the lowest lying state as the ground state in the negative parity sector. As in Refs. [20,17], masses from the projected correlation functions for the low-lying states are very consistent over the variational parameters of t_{start} and Δt, in particular, the negative parity ground state is robust. However, a deteriorating signal to noise is evident for larger t_{start} and Δt values, particularly for the excited states. In contrast, the mass from the eigenvalue analysis shows significant dependence on the variational parameters. Therefore, exposing a mass from the projected correlation functions is again proved to be more reliable than from the eigenvalues [17].

From a series of t_{start} and Δt, a single mass is selected for one set of t_{start} and Δt by the selection criteria discussed in Ref. [20], where we prefer larger value of $t_{\mathrm{start}}+\Delta t$ [21]. In cases where a larger $t_{\mathrm{start}}+\Delta t$ provides a poor signal-to-noise ratio, for example $(t_{\mathrm{start}}, \Delta t) = (7, 3)$ (top left graph of Fig. 1), we prefer a little lower $t_{\mathrm{start}}+\Delta t$ value, for example $(t_{\mathrm{start}}, \Delta t) = (7, 2)$, and we follow this procedure for each quark mass, as discussed in Ref. [18].

In Fig. 2, masses extracted from all the combinations of 3×3 matrices (from 1st to 5th) are shown for the pion mass of 797 and 380 MeV. Here the negative parity ground and the first excited states are very consistent for all the 3×3 bases. The second