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Spontaneous breaking of superconformal invariance in $(2+1)D$ supersymmetric Chern–Simons-matter theories in the large N limit

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ABSTRACT

In this work we study the spontaneous breaking of superconformal and gauge invariances in the Abelian $\mathcal{N}=1,2$ three-dimensional supersymmetric Chern–Simons-matter (SCSM) theories in a large N flavor limit. We compute the Kählerian effective superpotential at subleading order in $1/N$ and show that the Coleman–Weinberg mechanism is responsible for the dynamical generation of a mass scale in the $\mathcal{N}=1$ model. This effect appears due to two-loop diagrams that are logarithmic divergent. We also show that the Coleman–Weinberg mechanism fails when we lift from the $\mathcal{N}=1$ to the $\mathcal{N}=2$ SCSM model.

1. Introduction

The AdS/CFT correspondence which relates a special weak (strong) coupled string theory to a strong (weak) coupled superconformal field theory [1], opened a new freeway in the direction of the understanding of strong coupled gauge field theories. Several aspects of the correspondence have been studied [2,3]. In particular, the AdS_4/CFT_3 correspondence have attracted great attention in the literature due to its contribution for the development of the understanding of some condensed matter effects, especially the superfluidity [4] and the superconductivity [5,6]. Recently, Gaiotto and Yin suggested that various $\mathcal{N}=2,3$ three-dimensional SCSM theories are dual to open or closed string theories in AdS_4 [7]. These SCSM model are superconformal invariants, an essential ingredient to relate them to M2 branes [8–10].

On the other hand, it is known that in a three-dimensional non-supersymmetric Chern–Simons-matter theory the conformal symmetry is dynamically broken [11] by the Coleman–Weinberg mechanism [12] in two loop approximation; the same is also true for the superconformal invariance of the Abelian, $D=2+1$, $\mathcal{N}=1$ SCSM model [13], after two loops corrections to the effective (super) potential. For the $\mathcal{N}=2$ model, on the other hand, this mechanism fails to induce a breakdown of this symmetry.

In this work we study the spontaneous breaking of the superconformal and gauge invariances in the three-dimensional Abelian $\mathcal{N}=1,2$ SCSM theories in the large N flavor limit approximation. In Section 2 it is shown that the dynamical breaking of superconformal and gauge invariances in the $\mathcal{N}=1$ SCSM model is compatible with $1/N$ expansion, determining that the matter self-interaction coupling constant λ must be of the order of g^6/N, while no restriction to the gauge coupling g has to be imposed. In Section 3, it is discussed that similarly to what happens in the perturbative approach [13] the Coleman–Weinberg mechanism in the $1/N$ expansion is not feasible for the $\mathcal{N}=2$ extension of SCSM model. This happens because the coupling constants are constrained by the conditions that minimize the effective superpotential. In Section 4 the last comments and remarks are presented.

2. $\mathcal{N}=1$ SUSY Chern–Simons-matter model

The $\mathcal{N}=1$ three-dimensional supersymmetric Chern–Simons-matter model (SCSM) is defined by the classical action

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$$S=\int d^5z\left\{-\frac{1}{2}\Gamma^\alpha W_\alpha-\frac{1}{2}\overline{\nabla^\alpha\Phi_a}\nabla_\alpha\Phi_a+\lambda(\bar\Phi_a\Phi_a)^2\right\},\tag{1}$$

where $W^\alpha=(1/2)D^\beta D^\alpha\Gamma_\beta$ is the gauge superfield strength with Γ_β being the gauge superfield, $\nabla^\alpha=(D^\alpha-ig\Gamma^\alpha)$ is the supercovariant derivative, and a is an index that assume values from 1 to N, where N is the number of flavors of the complex superfields Φ. We use the notations and conventions as in [14]. When a mass term $\mu(\bar\Phi_a\Phi_a)$, with $\mu>0$, is present in the matter sector, the SCSM model exhibits spontaneous breaking of gauge invariance and a consequent generation of mass for the scalar and gauge superfields at tree level [15].

We are dealing with a classically superconformal model, and our aim in this work is to look for the possibility of dynamical breaking of the superconformal and gauge invariances in the $1/N$ expansion. To do this, let us redefine our coupling constants, $\lambda\to\frac{\lambda}{N}$, $g\to\frac{g}{\sqrt{N}}$, and shift the N-th component of the set of superfields Φ_a $(\bar\Phi_a)$ by the classical background superfield $\sigma_{cl}=\sigma_1-\theta^2\sigma_2$ as follows

$$\begin{aligned}\bar\Phi_N&=\frac{1}{\sqrt2}\left(\Sigma+\sqrt{N}\sigma_{cl}-i\Pi\right),\\ \Phi_N&=\frac{1}{\sqrt2}\left(\Sigma+\sqrt{N}\sigma_{cl}+i\Pi\right),\end{aligned}\tag{2}$$

with the vacuum expectation values (VEV) of the quantum superfields, i.e., $\langle\Sigma\rangle=\langle\Pi\rangle=\langle\Phi_j\rangle=0$ vanishing at any order of $1/N$. The index j runs over: $j=1,2,\dots,(N-1)$. To investigate the possibility of spontaneous breaking of gauge/superconformal symmetry is enough to obtain the Kählerian superpotential [13,16], i.e., to consider the contributions to the superpotential, where supersymmetric derivatives (D^α,D^2) acts only on the background superfield σ_{cl}.

The action written in terms of the real quantum superfields Σ and Π and the $(N-1)$ complex superfields Φ_j with vanishing VEVs is given by

$$\begin{aligned}S=\int d^5z\Big\{&-\frac{1}{2}\Gamma^\alpha W_\alpha-\frac{g^2\sigma_{cl}^2}{2}\Gamma^2+\frac{g}{2}\left(\sigma_{cl}D^\alpha\Pi\Gamma_\alpha+\Pi\Gamma_\alpha D^\alpha\sigma_{cl}\right)+\bar\Phi_j\left(D^2+\lambda\sigma_{cl}^2\right)\Phi_j+\frac{1}{2}\Sigma\left(D^2+3\lambda\sigma_{cl}^2\right)\Sigma\\ &+\frac{1}{2}\Pi\left(D^2+\lambda\sigma_{cl}^2\right)\Pi+i\frac{g}{2\sqrt{N}}\left(D^\alpha\bar\Phi_j\Gamma_\alpha\Phi_j+\bar\Phi_j\Gamma_\alpha D^\alpha\Phi_j\right)+\frac{g}{2\sqrt{N}}\left(D^\alpha\Pi\Gamma_\alpha\Sigma+\Pi\Gamma_\alpha D^\alpha\Sigma\right)\\ &-\frac{g^2}{2N}\left(2\bar\Phi_j\Phi_j+\Sigma^2+\Pi^2\right)\Gamma^2+\frac{\lambda}{N}(\bar\Phi_j\Phi_j)^2+\frac{\lambda}{4N}\left(\Sigma^2+\Pi^2\right)^2+\frac{\lambda}{N}\left(\Sigma^2+\Pi^2\right)\bar\Phi_j\Phi_j\\ &+\frac{\lambda}{\sqrt{N}}\sigma_{cl}\Sigma\left(2\bar\Phi_j+\Sigma^2+\Pi^2-\frac{g^2}{\lambda}\Gamma^2\right)+\sqrt{N}\left(\lambda\sigma_{cl}^3+D^2\sigma_{cl}\right)\Sigma+N\sigma_{cl}D^2\sigma_{cl}+N\frac{\lambda}{4}\sigma_{cl}^4\\ &-\frac{1}{4\alpha}\left(D^\alpha\Gamma_\alpha+\alpha g\sigma_{cl}\Pi\right)^2+\bar cD^2c+\alpha\frac{g^2\sigma_{cl}^2}{2}\bar cc+\frac{\alpha}{2\sqrt{N}}g^2\sigma_{cl}\bar c\Sigma c+\mathcal{L}_{ct}\Big\},\end{aligned}\tag{3}$$

where the last line of above equation is the R_ξ gauge-fixing term and the corresponding Faddeev–Popov terms, plus counterterms of renormalization represented by $\mathcal{L}_{ct}$. The term $\frac{-g\sigma_{cl}}{2}D^\alpha\Pi\Gamma_\alpha$ is responsible for the mixing between the scalar superfield Π and the gauge superfield Γ^α. The introduction of an R_ξ gauge-fixing eliminate this mixing, in the approximation considered.

From the action above, Eq. (3), we can compute the free propagators, Fig. 1, of the model as

$$\begin{aligned}&\left\langle T\Phi_i(k,\theta)\bar\Phi_j\left(-k,\theta'\right)\right\rangle=-i\delta_{ij}\frac{D^2-M_0}{k^2+M_0^2}\delta^{(2)}\left(\theta-\theta'\right),\\ &\left\langle T\Sigma(k,\theta)\Sigma\left(-k,\theta'\right)\right\rangle=-i\frac{D^2-M_1}{k^2+M_1^2}\delta^{(2)}\left(\theta-\theta'\right),\\ &\left\langle T\Pi(k,\theta)\Pi\left(-k,\theta'\right)\right\rangle=-i\frac{D^2-M_2}{k^2+M_2^2}\delta^{(2)}\left(\theta-\theta'\right),\\ &\left\langle T\Gamma_\alpha(k,\theta)\Gamma_\beta\left(-k,\theta'\right)\right\rangle=-\frac{i}{2}\left[\frac{(D^2-M_A)D^2D_\beta D_\alpha}{k^2(k^2+M_A^2)}-\alpha\frac{(D^2-\alpha M_A)D^2D_\alpha D_\beta}{k^2(k^2+\alpha^2M_A^2)}\right]\delta^{(2)}\left(\theta-\theta'\right),\\ &\left\langle Tc(k,\theta)\bar c\left(-k,\theta'\right)\right\rangle=-i\frac{D^2+\alpha M_A}{k^2+\alpha^2M_A^2}\delta^{(2)}\left(\theta-\theta'\right),\end{aligned}\tag{4}$$

where

$$M_0=\lambda\sigma_{cl}^2,\qquad M_1=3\lambda\sigma_{cl}^2,\qquad M_A=\frac{g^2\sigma_{cl}^2}{2},\qquad M_2=\lambda\sigma_{cl}^2-\frac{\alpha}{2}M_A.\tag{5}$$

It is important to notice that these propagators are obtained as an approximation, where we are neglecting any superderivative acting on background superfield σ_{cl}. This approximation is the enough to obtain the three-dimensional Kählerian effective superpotential, as described in [17]. It does not allow us to evaluate the higher order quantum corrections of the auxiliary field σ_2. One way to do this, is to approach the effective superpotential by using the component formalism, as was done in the Wess–Zumino model in [18]. Even though our aim is to study the SCSM model in the large N limit, one more approximation will be considered: we will restrict to small values of the coupling λ, a choice to be justified later, when we will show that λ must be of the order of g^6/N.