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Weak basis transformations and texture zeros in the leptonic sector

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ABSTRACT

We investigate the physical meaning of some of the texture zeros which appear in most of the ansatzes on leptonic masses and their mixing. It is shown that starting from arbitrary lepton mass matrices and making suitable weak basis transformations one can obtain some of these sets of zeros, which therefore have no physical content. We then analyse four-zero texture ansatzes where the charged lepton and neutrino mass matrices have the same structure. The four texture zeros cannot be obtained simultaneously through weak basis transformations, so these ansatzes do have physical content. We show that they can be separated into four classes and study the physical implications of each class.

1. Introduction

The discovery of neutrino oscillations pointing towards the existence of non-vanishing neutrino masses and large leptonic mixing has rendered the flavour puzzle even more intriguing. There have been many attempts at understanding the pattern of leptonic masses and mixing [1], including the introduction of either Abelian or non-Abelian flavour symmetries, some of them leading to texture zeros in the fermion mass matrices. In the leptonic sector there is an extra motivation for introducing texture zeros, namely the fact that without an appeal to theory, it is not possible to fully reconstruct the neutrino mass matrix m_ν from experimental input arising from feasible experiments. It has been shown that this is possible if one postulates the presence of texture zeros in m_ν [2] or if one assumes that $\det(m_\nu)$ vanishes [3].

A difficulty one encounters in an attempt at making a systematic study of experimentally viable texture zeros results from the fact that some sets of these zeros have, by themselves, no physical meaning, since they can be obtained starting from arbitrary fermion mass matrices, by making appropriate weak basis (WB) transformations which leave the gauge currents flavour diagonal [4].

In this Letter we investigate in detail what are the texture zeros which can be obtained in the leptonic sector with Majorana neutrinos through WB transformations. We then analyse the physical implications of ansatzes where the charged lepton mass matrix m_ℓ and the effective Majorana neutrino mass matrix m_ν have the same structure (we denote them "parallel ansatzes"), with a total of four independent zeros. These ansatzes do have physical meaning, since not all their texture zeros can be simultaneously obtained through WB transformations. Although there is no universal principle requiring parallel structures, they certainly have an aesthetical appeal and naturally arise in some classes of family symmetries as well as in the framework of some grand-unified theories [5].

This Letter is organised as follows. In the next section, we show that starting from arbitrary structures for the leptonic mass matrices it is possible to obtain, through WB transformations, m_ℓ Hermitian with a texture zero in the $(1,1)$ position while m_ν (which is symmetric due to its assumed Majorana nature) has zeros in the $(1,1)$ and $(1,3)$ entries. In Section 3, we analyse four-zero parallel ansatzes, showing that they can be divided into four different classes. In Section 4 we confront these ansatzes with the present experimental data and analyse their predictions. Finally, in the last section we draw our conclusions.

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2. Creating texture zeros through WB transformations

We assume the Standard Model with left-handed neutrinos together with some unspecified mechanism leading to lepton-number violation and the generation of a left-handed Majorana mass for neutrinos. The most general WB transformation which leaves the gauge currents invariant is

$$m_\ell \rightarrow m'_\ell = W^\dagger m_\ell W_R, \qquad m_\nu \rightarrow m'_\nu = W^{\mathrm{T}} m_\nu W, \tag{1}$$

where W and W_R are 3×3 unitary matrices, while m_ℓ, m_ν denote the charged lepton and neutrino mass matrices, respectively. It is possible to make a WB transformation which renders m_ℓ real and diagonal. In this basis, one has:

$$m_\ell = D_\ell, \qquad m_\nu = U^* D_\nu U^\dagger, \tag{2}$$

where $D_\ell = \mathrm{diag}(m_e, m_\mu, m_\tau)$ and $D_\nu = \mathrm{diag}(m_1, m_2, m_3)$ are real diagonal matrices. The unitary matrix U is the so-called Pontecorvo–Maki–Nakagawa–Sakata (PMNS) matrix [6] which can be parametrised as

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{1}3e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} P_\alpha, \tag{3}$$

where $c_{ij} \equiv \cos\theta_{ij}$, $s_{ij} \equiv \sin\theta_{ij}$; θ_{ij} are mixing angles and δ is CP-violating Dirac phase. The diagonal matrix $P_\alpha = \mathrm{diag}(e^{i\alpha_1/2}, e^{i\alpha_2/2}, 1)$ contains the Majorana phases, α_1, α_2, which have physical meaning only if light neutrinos are Majorana particles. Although Majorana phases do not affect neutrino oscillations, they do play a role in neutrinoless double beta decay, contributing to so-called effective Majorana mass [7]

$$m_{\beta\beta} \equiv m_1 U_{e1}^{*2} + m_2 U_{e2}^{*2} + m_3 U_{e3}^{*2}. \tag{4}$$

2.1. Creating the $(1,1)$ zero in m_ℓ and m_ν

Our goal is to investigate whether it is always possible to find a WB transformation which, starting from arbitrary matrices m_ℓ and m_ν, in the basis given in Eq. (2), leads to new matrices m'_ℓ and m'_ν such that $(m'_\ell)_{11} = (m'_\nu)_{11} = 0$ and m_ℓ Hermitian. In this case, the WB transformations of Eq. (1) are restricted to those with $W_R = W$, i.e.,

$$m_\ell \rightarrow m'_\ell = W^\dagger D_\ell W, \qquad m_\nu \rightarrow m'_\nu = W^{\mathrm{T}} U^* D_\nu U^\dagger W. \tag{5}$$

The requirement that $(m'_\ell)_{11}$ and $(m'_\nu)_{11}$ vanish leads to the conditions

$$m_e|W_{11}|^2 + m_\mu|W_{21}|^2 + m_\tau|W_{31}|^2 = 0, \tag{6}$$

$$m_1 X_{11}^2 + m_2 X_{21}^2 + m_3 X_{31}^2 = 0, \tag{7}$$

where $X \equiv U^\dagger W$. The matrix elements X_{i1}^2 $(i = 1, 2, 3)$ in Eq. (7) are given by

$$X_{i1}^2 = U_{1i}^{*2} W_{11}^2 + U_{2i}^{*2} W_{21}^2 + U_{3i}^{*2} W_{31}^2 + 2\,U_{1i}^* W_{11} U_{2i}^* W_{21} + 2\,U_{1i}^* W_{11} U_{3i}^* W_{31} + 2\,U_{2i}^* W_{21} U_{3i}^* W_{31}. \tag{8}$$

It is clear that in order for Eq. (6) to have a solution, one of the masses m_e, m_μ or m_τ must have a sign opposite to the other two. This requirement can be always fulfilled, since the sign of a Dirac fermion mass can always be changed by making an appropriate chiral transformation. In order for Eq. (7) to have a solution, the three real non-negative quantities $a_i \equiv |m_i X_{i1}^2|$ should be such that a triangle can be formed with sides a_1, a_2 and a_3. A necessary and sufficient condition for them to be the sides of a triangle is that:

$$2\left(a_1^2 a_2^2 + a_1^2 a_3^2 + a_2^2 a_3^2\right) - a_1^4 - a_2^4 - a_3^4 \geqslant 0. \tag{9}$$

Given (m_e, m_μ, m_τ), (m_1, m_2, m_3) and U, a solution to Eqs. (6) and (7) can be found through the following procedure:

(i) Find $|W_{11}|$, $|W_{21}|$ and $|W_{31}|$ such that Eq. (6) is satisfied. It is clear that this is always possible. One can parametrise the first column of W as

$$|W_{11}| = \cos\theta\cos\psi, \qquad |W_{21}| = \sin\theta\cos\psi, \qquad |W_{31}| = \sin\psi. \tag{10}$$

Then a solution of Eq. (6) can be found by adjusting the angles θ and ψ.

(ii) In order to satisfy Eq. (7), one has to choose W in such a way that the inequality in Eq. (9) is verified. Finding a solution of Eq. (7) is then equivalent to the problem of determining the internal angles of a triangle from the knowledge of its sides. If we denote $\varphi_{ij} \equiv \arg(X_{ij})$, the internal angles of the triangle are given by $2(\varphi_{21} - \varphi_{11})$ and $2(\varphi_{31} - \varphi_{11})$.

2.2. Creating an additional zero

Once the zero in the position $(1,1)$ is obtained, a natural question to ask is whether one can get additional WB zeros while keeping m_ℓ Hermitian. It can be readily seen that there exists a second WB transformation that keeps $(m'_\ell)_{11} = (m'_\nu)_{11} = 0$ and leads either to $(m'_\ell)_{13} = 0$ or to $(m'_\nu)_{13} = 0$. Such a transformation is defined by the unitary matrix

$$W = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -e^{i\varphi}\sin\theta \\ 0 & e^{-i\varphi}\sin\theta & \cos\theta \end{pmatrix}, \tag{11}$$