Physics Letters B 672 (2009) 264–269

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Relativistic description of the double charmonium production in e^+e^- annihilation

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ARTICLE INFO

Article history:
Received 25 March 2008
Received in revised form 16 January 2009
Accepted 16 January 2009
Available online 21 January 2009
Editor: A. Ringwald

PACS:
13.66.Bc
12.39.Ki
12.38.Bx

Keywords:
Hadron production in e^+e^- interactions
Relativistic quark model

ABSTRACT

New evaluation of the relativistic effects in the double production of S-wave charmonium states is performed on the basis of perturbative QCD and the relativistic quark model. The main improvement consists in the exact account of properties of the relativistic meson wave functions. For the gluon and quark propagators entering the production vertex function we use a truncated expansion in the ratio of the relative quark momenta to the center-of-mass energy $\sqrt{s}$ up to the second order. The exact relativistic treatment of the wave functions makes all such second order terms convergent, thus allowing the reliable calculation of their contributions to the production cross section. Compared to the nonrelativistic calculation we obtain a significant increase of the cross sections for the S-wave double charmonium production. This brings new theoretical results in good agreement with the available experimental data.

The production processes of mesons and baryons containing heavy b and c quarks in different reactions are under intensive study at present [1–4]. The experimental investigation of the double charmonium production in e^+e^- annihilation by BaBar and Belle Collaborations revealed a discrepancy between the measured cross sections and theoretical results obtained in the nonrelativistic approximation in QCD [5–7]. Various efforts have been undertaken to improve the theoretical calculations. They include the evaluation of radiative corrections of order α_s and the investigation of relativistic effects connected with the relative motion of the heavy quarks forming the vector and pseudoscalar quarkonia [7–19]. As a result, the difference between theory and experiment for the value of the center-of-mass energy $\sqrt{s} = 10.6$ GeV was essentially decreased [7,12,13,15]. Moreover, the new theoretical analysis carried out in Refs. [19,20] shows that the inclusion of order α_s and relativistic corrections decreases the discrepancy between theory and experiment at the present level of precision. But despite this fact there exists the frequently debated question connected with the calculation of the relativistic corrections in the production cross section. It is related to the determination of the specific parameter $\langle \mathbf{p}^2\rangle = \int \mathbf{p}^2 \Psi_0^{\mathcal{P},\mathcal{V}}(\mathbf{p})\, d\mathbf{p}/(2\pi)^3$ emerging after the expansion of all quantities in the production amplitude in the relative quark momenta $\mathbf{p}$ and $\mathbf{q}$ [15,20–22], where $\Psi_0^{\mathcal{V},\mathcal{P}}$ are the vector and pseudoscalar charmonium wave functions in the rest frame. The divergence of this integral required the use of a regularization procedure (dimensional regularization is commonly used) which led to a definite uncertainty of the evaluation. Moreover, the large value of the relativistic contribution obtained in the previous studies [7,15] evidently rises a question about the convergence of the expansion in the heavy quark velocity. In this Letter we propose an alternative approach to the calculation of relativistic effects based on the relativistic quark model [23–27] and perturbative QCD. It uses a truncated expansion in relative momenta $\mathbf{p}$ and $\mathbf{q}$ and thus avoids divergent integrals in the relativistic contribution of the second order.

Define the four momenta of the produced $c, \bar{c}$ quarks forming the vector and pseudoscalar charmonia in terms of total momenta $P(Q)$ and relative momenta $p(q)$ as follows:

$$p_{1,2} = \frac{1}{2}P \pm p, \quad (p\cdot P) = 0, \qquad q_{1,2} = \frac{1}{2}Q \pm q, \quad (q\cdot Q) = 0, \tag{1}$$

where $p = L_P(0,\mathbf{p})$, $q = L_P(0,\mathbf{q})$ are the four-momenta obtained from the rest frame four-momenta $(0,\mathbf{p})$ and $(0,\mathbf{q})$ by the Lorentz transformation to the system moving with the momenta P, Q. Then the production amplitude of the S-wave vector and pseudoscalar

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0370-2693
doi:10.1016/j.physletb.2009.01.029

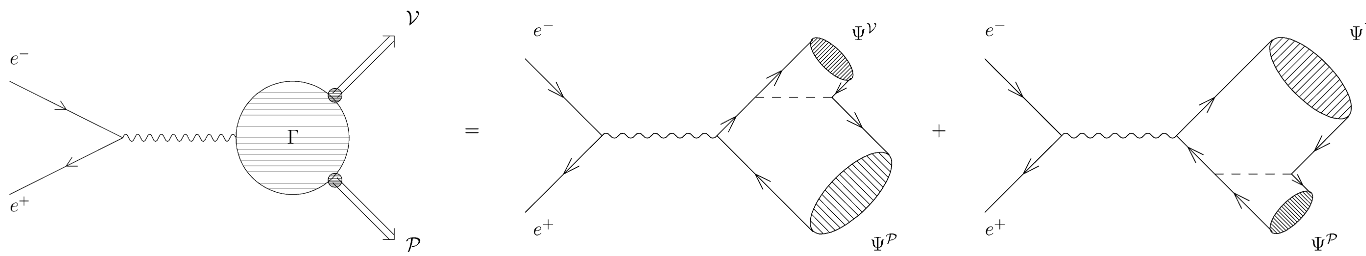

Fig. 1. The production amplitude of a pair of charmonium states ($\mathcal{V}$ denotes the vector meson and $\mathcal{P}$ the pseudoscalar meson) in e^+e^- annihilation. The wave line shows the virtual photon and the dashed line corresponds to the gluon. Γ is the production vertex function.

charmonium states, shown in Fig. 1, can be presented in the form [15,28,29]:

$$\mathcal{M}(p_-, p_+, P, Q) = \frac{8\pi^2\alpha\alpha_s\mathcal{Q}_c}{3s}\bar v(p_+)\gamma^\beta u(p_-)\int\frac{d\mathbf{p}}{(2\pi)^3}\int\frac{d\mathbf{q}}{(2\pi)^3}Sp\{\Psi^{\mathcal{V}}(p,P)\Gamma^\nu(p,q,P,Q)\Psi^{\mathcal{P}}(q,Q)\gamma_\nu\}, \tag{2}$$

where α_s is the QCD coupling constant, α is the fine structure constant, $\mathcal{Q}_c$ is the c quark electric charge. The relativistic wave functions of the bound quarks $\Psi^{\mathcal{V},\mathcal{P}}$ accounting for the transformation from the rest frame to the moving one with four momenta P, Q are

$$\Psi^{\mathcal{V}}(p,P) = \frac{\Psi_0^{\mathcal{V}}(\mathbf{p})}{[\frac{\epsilon(p)}{m}\frac{(\epsilon(p)+m)}{2m}]}\left[\frac{\hat v_1-1}{2}+\hat v_1\frac{\mathbf{p}^2}{2m(\epsilon(p)+m)}-\frac{\hat p}{2m}\right]\hat{\tilde\epsilon}^*(1+\hat v_1)\left[\frac{\hat v_1+1}{2}+\hat v_1\frac{\mathbf{p}^2}{2m(\epsilon(p)+m)}+\frac{\hat p}{2m}\right], \tag{3}$$

$$\Psi^{\mathcal{P}}(q,Q) = \frac{\Psi_0^{\mathcal{P}}(\mathbf{q})}{[\frac{\epsilon(q)}{m}\frac{(\epsilon(q)+m)}{2m}]}\left[\frac{\hat v_2-1}{2}+\hat v_2\frac{\mathbf{q}^2}{2m(\epsilon(q)+m)}+\frac{\hat q}{2m}\right]\gamma_5(1+\hat v_2)\left[\frac{\hat v_2+1}{2}+\hat v_2\frac{\mathbf{q}^2}{2m(\epsilon(q)+m)}-\frac{\hat q}{2m}\right], \tag{4}$$

where $v_1 = P/M_{\mathcal{V}}$, $v_2 = Q/M_{\mathcal{P}}$; $\tilde\epsilon$ is the polarization vector of the vector charmonium; $\epsilon(p) = \sqrt{p^2+m^2}$ and m is the c quark mass. The vertex function $\Gamma^\nu(p,P;q,Q)$ at leading order in α_s can be written as a sum of four contributions:

$$\begin{aligned}\Gamma^\nu(p,P;q,Q) = {} & \gamma_\mu\frac{(\hat r-\hat q_1+m)}{(r-q_1)^2-m^2+i\epsilon}\gamma_\beta D^{\mu\nu}(k_2)+\gamma_\beta\frac{(\hat p_1-\hat r+m)}{(r-p_1)^2-m^2+i\epsilon}\gamma_\mu D^{\mu\nu}(k_2)\\ & +\gamma_\beta\frac{(\hat q_2-\hat r+m)}{(r-q_2)^2-m^2+i\epsilon}\gamma_\mu D^{\mu\nu}(k_1)+\gamma_\mu\frac{(\hat r-\hat p_2+m)}{(r-p_2)^2-m^2+i\epsilon}\gamma_\beta D^{\mu\nu}(k_1),\end{aligned} \tag{5}$$

where the gluon momenta are $k_1 = p_1+q_1$, $k_2 = p_2+q_2$ and $r^2 = s = (P+Q)^2 = (p_-+p_+)^2$, p_-, p_+ are four momenta of the electron and positron. The dependence on the relative momenta of c-quarks is present both in the gluon propagator $D_{\mu\nu}(k)$ and quark propagators as well as in the relativistic wave functions. One of the main technical difficulties in calculating the production amplitude (2) consists in performing angular integrations, since both gluon and quark propagators in the vertex function (5) contain angles in the denominators. Therefore we expand these propagators in the relative momenta. Such expansion leads to the vertex function containing angles only in numerators and, thus, the angular integrations can be easily performed.

The inverse denominators of quark propagators expanded in the ratio of the relative quark momenta p, q to the energy $\sqrt{s}$ up to the second order can be expressed as follows:

$$\frac{1}{(r-q_{1,2})^2-m^2} = \frac{1}{Z_1}\left[1-\frac{q^2}{Z_1}\pm\frac{2(rq)}{Z_1}+\frac{4(rq)^2}{Z_1^2}+\cdots\right], \tag{6}$$

$$\frac{1}{(r-p_{1,2})^2-m^2} = \frac{1}{Z_2}\left[1-\frac{p^2}{Z_2}\pm\frac{2(rp)}{Z_2}+\frac{4(rp)^2}{Z_2^2}+\cdots\right], \tag{7}$$

where the factors Z_1 and Z_2 differ only due to the bound state corrections:

$$Z_1 = \frac{2s+2M_{\mathcal{V}}^2-M_{\mathcal{P}}^2-4m^2}{4},\qquad Z_2 = \frac{2s+2M_{\mathcal{P}}^2-M_{\mathcal{V}}^2-4m^2}{4}. \tag{8}$$

Corresponding expansions of the gluon propagators in Eq. (5) with the account of terms of order $O(p^2/s, q^2/s)$ are ($Z = s/4$):

$$\frac{1}{k_{2,1}^2} = \frac{1}{Z}\left[1-\frac{p^2+q^2+2pq}{Z}\pm\frac{(rp)+(rq)}{Z}+\frac{(rp)^2+(rq)^2+2(rp)(rq)}{Z^2}+\cdots\right]. \tag{9}$$

We expanded the gluon and quark propagators in the ratio of the relative quark momenta to the center-of-mass energy $\sqrt{s}$ up to the second order terms in the production vertex function (5) but preserved all relativistic factors entering the denominators of the relativistic wave functions (3), (4). This provides the convergence of the resulting momentum integrals. Then keeping the terms of second and fourth order in both variables p and q in the numerator of Eq. (2) from the relativistic wave functions (3)–(4) and second order from the expansions of the quark and gluon propagators, we perform the angular averaging taking into account Eq. (1) and using the following relation:

$$\int p_\mu p_\nu\, d\Omega_{\mathbf{p}} = -\frac{1}{3}\mathbf{p}^2\left(g_{\mu\nu}-\frac{P_\mu P_\nu}{M^2}\right). \tag{10}$$

Then we can write the total production amplitude $\mathcal{M}$ in the form: