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Some physics of the two-dimensional $\mathcal{N} = (2,2)$ supersymmetric Yang–Mills theory: Lattice Monte Carlo study

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ABSTRACT

We illustrate some physical application of a lattice formulation of the two-dimensional $\mathcal{N} = (2,2)$ supersymmetric $SU(2)$ Yang–Mills theory with a (small) supersymmetry breaking scalar mass. Two aspects, power-like behavior of certain correlation functions (which implies the absence of the mass gap) and the static potential $V(R)$ between probe charges in the fundamental representation, are considered. For the latter, for $R \lesssim 1/g$, we observe a linear confining potential with a finite string tension. This confining behavior appears distinct from a theoretical conjecture that a probe charge in the fundamental representation is screened in two-dimensional gauge theory with an adjoint massless fermion, although the static potential for $R \gtrsim 1/g$ has to be systematically explored to conclude real asymptotic behavior in large distance.

1. Introduction

Recently, through the observation of a "partially conserved supercurrent relation", we obtained [1] an affirmative numerical evidence that a lattice formulation in Ref. [2] provides a supersymmetric regularization of the two-dimensional $\mathcal{N} = (2,2)$ supersymmetric Yang–Mills theory (SYM) [1] [2]

$$S = \frac{1}{g^2}\int d^2x\,\mathrm{tr}\left\{\frac{1}{2}F_{MN}F_{MN} + \Psi^T C\Gamma_M D_M\Psi + \tilde{H}^2\right\}, \tag{1}$$

when one supplements to S a supersymmetry breaking scalar mass term

$$S_{\text{mass}} = \frac{1}{g^2}\int d^2x\,\mu^2\,\mathrm{tr}\{A_2A_2 + A_3A_3\}. \tag{2}$$

The scalar mass term was added to suppress a possible large amplitude of scalar fields along flat directions that may amplify $O(a)$ lattice artifacts to $O(1)$ [1]. In the present Letter, we illustrate some physical application of this lattice formulation for the system $S + S_{\text{mass}}$.

2. Correlation functions with power-like behavior

Assuming the 't Hooft anomaly matching condition, in Ref. [20], it was pointed out that the two-dimensional $\mathcal{N} = (2,2)$ SYM has no mass gap. This aspect has been numerically investigated from almost a decade ago [21,22] by utilizing the supersymmetric discretized light-cone formulation [23]. In this super-renormalizable system, it is in fact possible to determine (to all orders of perturbation theory) an explicit form of a correlation function between Noether currents, by employing anomalous Ward–Takahashi (WT) identities (i.e. the Kac–Moody algebra) [24]; this explicit form directly proves the above assertion. Here, rather than supersymmetry, continuous global (bosonic) symmetries are important and the proof [24] applies even with supersymmetry breaking scalar mass term (2).

The total action $S + S_{\text{mass}}$ is invariant under the (two-dimensional) $U(1)_V$ transformation, $\Psi \to \exp\{i\alpha\Gamma_5\}\Psi$, and an associated Noether current ($U(1)_V$ current) is given by

$$j_\mu \equiv \frac{1}{g^2}\,\mathrm{tr}\{\Psi^T C\Gamma_\mu\Gamma_5\Psi\}. \tag{3}$$

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[1] For other lattice formulations of this system, see Refs. [3–9]. For recent developments in this field of research, see Ref. [10] for a review and references cited in Ref. [1]. As further recent study, see Refs. [11–19].

[2] This system can be obtained by dimensionally reducing the four-dimensional $\mathcal{N} = 1$ SYM from four to two dimensions and hence a four-dimensional notation is useful; Roman indices M and N run over 0, 1, 2 and 3, while Greek indices μ and ν below run over only 0 and 1. With the dimensional reduction, it is understood that $\partial_2 = 0$ and $\partial_3 = 0$. Ψ is a four-component spinor. We follow the notational convention in Ref. [1]. Note that the gauge coupling g has the mass dimension 1.

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Table 1
Sets of uncorrelated configurations used for Figs. 1 and 2. The scalar mass squared is $\mu^2/g^2 = 0.25$ for all cases.

Lattice size	ag	$\beta g \times Lg$	Number of configurations	Set label
16×8	0.1768	2.828×1.414	400	I
20×10	0.1414	2.828×1.414	800	II
24×12	0.1179	2.828×1.414	400	III
20×16	0.1414	2.828×2.263	400	IV

Similarly, associated with the $U(1)_A$ symmetry, $\Psi \to \exp\{\alpha \Gamma_2 \Gamma_3\}\Psi$, $A_2 \to \cos\{2\alpha\}A_2 - \sin\{2\alpha\}A_3$ and $A_3 \to \sin\{2\alpha\}A_2 + \cos\{2\alpha\}A_3$, there is a Noether current ($U(1)_A$ current),

$$j_{5\mu} \equiv \frac{1}{g^2} \operatorname{tr}\bigl\{-i\Psi^T C \Gamma_\mu \Gamma_2 \Gamma_3 \Psi + 4i(A_3 F_{\mu 2} - A_2 F_{\mu 3})\bigr\}. \tag{4}$$

It is then possible to show that [24], for the two-dimensional euclidean space $\mathbb{R}^2$,

$$\begin{aligned} &-\frac{i}{2}\langle j_\mu(x)\epsilon_{\nu\rho} j_{5\rho}(0)\rangle \\ &\quad = \frac{1}{4\pi}\bigl(N_c^2 - 1\bigr) \int \frac{d^2 p}{(2\pi)^2} e^{ipx} \\ &\qquad \times \left\{ -\frac{1}{p^2}(p_\mu p_\nu - \epsilon_{\mu\rho}\epsilon_{\nu\sigma} p_\rho p_\sigma) + \tilde{c}\delta_{\mu\nu} \right\} \\ &\quad = \frac{1}{4\pi}\bigl(N_c^2 - 1\bigr) \left\{ \frac{1}{\pi}\frac{1}{(x^2)^2}(x_\mu x_\nu - \epsilon_{\mu\rho}\epsilon_{\nu\sigma} x_\rho x_\sigma) + \tilde{c}\delta_{\mu\nu}\delta^2(x) \right\}, \end{aligned} \tag{5}$$

to all orders of perturbation theory, where N_c is the number of colors and the constant $\tilde{c}$ is a regularization ambiguity in a divergent one-loop diagram. Thus the correlation function between the $U(1)_V$ current and the $U(1)_A$ current possesses a massless pole and this is precisely what the 't Hooft anomaly matching condition claims for this two-dimensional system.

We want to confirm the power-like behavior of correlation function in Eq. (5) by using a lattice Monte Carlo simulation. For this, we prepared sets of uncorrelated configurations listed in Table 1. For simulation details, see Refs. [1,25,26]. In the table, a denotes the lattice spacing and β and L are temporal and spatial physical sizes of our lattice, respectively. The scalar mass squared is $\mu^2/g^2 = 0.25$ for all cases. The temporal boundary condition for fermionic variables is antiperiodic as in Ref. [1]. For current operator (4), we discretized the covariant derivatives $F_{\mu 2} = D_\mu A_2$ and $F_{\mu 3} = D_\mu A_3$ by using the forward covariant lattice difference. Eq. (5) suggests that we should not take an average of the correlation function over the spatial coordinate x_1 (i.e., projection to the zero spatial momentum) because after the average, correlation function (5) becomes proportional to $\delta(x_0)$ that cannot be distinguished from the regularization ambiguity; we should measure the correlation function as it stands without the zero spatial momentum projection.

In Fig. 1, we plotted $-i\langle j_\mu(x)\epsilon_{\nu\rho} j_{5\rho}(0)\rangle/2$ with $\mu = \nu = 0$ along the line $x_1 = 0$. We plotted also theoretical prediction (5) for $\mathbb{R}^2$ with $N_c = 2$, $(3/4\pi^2)1/(x_0)^2$, by the broken line. We clearly see the power-like fall of the correlation function for $x_0 g \lesssim 0.7$[3] instead of exponential one, although the overall amplitude is somewhat larger than the theoretical expectation for $\mathbb{R}^2$. From the behavior in the figure, we think that this discrepancy in the overall amplitude is caused by a finite lattice spacing and volume. In particular, comparison between set II (indicated by $\times$) and set IV (indicated by $\bigcirc$) shows that the finite size effect is rather large (note that these two sets differ only in the spatial physical size L). We thus expect that the theoretical prediction for $\mathbb{R}^2$ is eventually reproduced in the limit, $a \to 0$ and β, $L \to \infty$, although we do not carry out a systematic study on this limit.

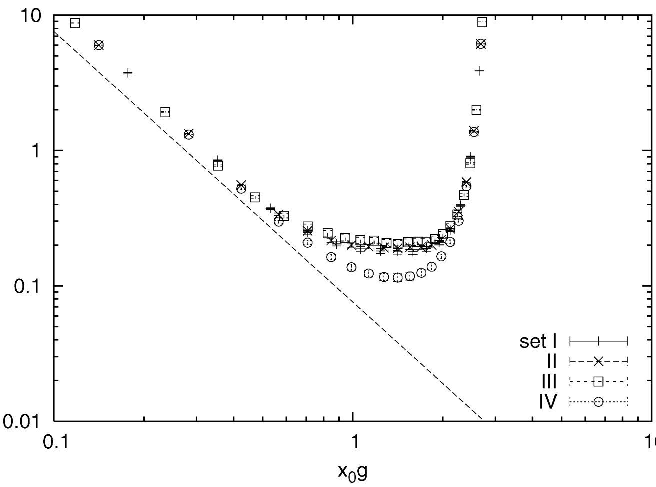

Fig. 1. The correlation function $-i\langle j_\mu(x)\epsilon_{\nu\rho} j_{5\rho}(0)\rangle/(2g^2)$ with $\mu = \nu = 0$ along the line $x_1 = 0$, for the configuration sets in Table 1. The broken line is theoretical prediction (5) for $\mathbb{R}^2$.

What is the implication of the above observation? It indicates that our target theory, the two-dimensional $\mathcal{N} = (2,2)$ $SU(2)$ SYM with a scalar mass term, is realized in the continuum limit of the present lattice model. In particular, in deriving Eq. (5), one assumes that the $U(1)_V$ and $U(1)_A$ currents j_μ and $j_{5\nu}$ individually conserve [24].[4] One assumes $U(1)_V$ and $U(1)_A$ symmetries in this sense. In the present lattice formulation [2], the $U(1)_V$ symmetry is explicitly broken for finite lattice spacings. The above observation hence indicates that the $U(1)_V$ symmetry is fairly restored with present lattice spacings. (This symmetry will eventually be restored in the continuum limit [1].)

Now, if the system were supersymmetric, and if supersymmetry is not spontaneously broken, there would exist a massless fermionic state corresponding to the massless bosonic state appearing in Eq. (5) as an intermediate state. We expect that this fermionic state produces a massless pole in the correlation functions

$$\bigl\langle (s_\mu)_i(x)(f_\nu)_i(0)\bigr\rangle \quad (i = 1, 2, 3, 4;\ \text{no sum over } i), \tag{6}$$

where i denotes the spinor index and

$$s_\mu \equiv -\frac{1}{g^2} C \Gamma_M \Gamma_N \Gamma_\mu \operatorname{tr}\{F_{MN}\Psi\}, \tag{7}$$

$$f_\mu \equiv \frac{1}{g^2}\Gamma_\mu\bigl(\Gamma_2 \operatorname{tr}\{A_2\Psi\} + \Gamma_3 \operatorname{tr}\{A_3\Psi\}\bigr). \tag{8}$$

In the above, s_μ is the supercurrent associated with the supersymmetry of S, $\delta A_M = i\epsilon^T C \Gamma_M \Psi$, $\delta\Psi = \frac{i}{2} F_{MN}\Gamma_M\Gamma_N\epsilon + i\tilde{H}\Gamma_5\epsilon$, and $\delta\tilde{H} = -i\epsilon^T C \Gamma_5 \Gamma_M D_M \Psi$, and f_μ is a lowest-dimensional fermionic spinor-vector (considered in Ref. [1]). Eq. (6) with $i = 1$, 2, 3, and 4 are precisely four correlation functions studied in Eq. (11) of Ref. [1] and, as noted there, these four functions are identical to

[3] In Fig. 1, we plotted the correlation function as a function of x_0, along the line $x_1 = 0$. As x_0 moves away from the origin $x_0 = 0$, the point x approaches a periodic image of the origin at $x_0 = \beta$ and for $x_0 \gtrsim \beta/2$ we expect the correlation function is power-like in the variable $\beta - x_0$. In other words, the fact that our finite-size lattice is topologically T^2 but not $\mathbb{R}^2$ cannot be neglected for $x_0 \gtrsim \beta/2$. We thus do not expect the power-like fall (that is expected for $\mathbb{R}^2$) for $x_0 g \gtrsim 1$ and actually the plot blows up for $x_0 g \gtrsim 1$ (in our simulation, $\beta g = 2.828$). This remark is applied also to Fig. 2, in which the antiperiodic boundary condition for fermionic fields implies "blow-down" for $x_0 g \gtrsim 1$.

[4] This assumption fails, for example, in the massless Schwinger model, in which the $U(1)_A$ current suffers from the axial anomaly; note that the massless Schwinger model has a mass gap.