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Fermions tunnelling from GHS and non-extremal D1–D5 black holes

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ABSTRACT

Recent research shows that fermions tunnelling can result in correct Hawking temperature of a black hole. In this letter, choosing a set of appropriate matrices γ^{μ}, we attempt to study Hawking radiation of Dirac particles across the horizons of the GHS and non-extremal five-dimensional D1–D5 black holes in string theory by using fermions tunnelling method. Finally, the expected Hawking temperatures of the GHS and non-extremal D1–D5 black holes are correctly recovered.

1. Introduction

In 1974, Hawking proved that a black hole can radiate particles with thermal spectrum at the temperature $T = \kappa/2\pi$ [1], where κ is the surface gravity of the horizon. Since then, although many methods have appeared to correctly derive Hawking radiation of a black hole, not all of them are satisfying in the process of extending these methods. In 2000, Parikh and Wilczek, elaborating Kraus and Wilczek's work [2,3], presented another new derivation of Hawking radiation, where Hawking radiation is treated as a quantum tunnelling process [4]. A pair of particles is spontaneously created just inside the horizon as a result of quantum vacuum fluctuations near the horizon, classically the positive energy particle do not escape out along the classically forbidden region, but quantum mechanically it can tunnel out to the infinity. For the tunnelling picture, the Hawking temperature is directly related to the imaginary part of the action of particles tunnelling from inside to outside horizon along the classically forbidden region. In [4], derivation of the imaginary part of the action mainly depends on the integration of the radial momentum p_r for the emitted particles, which is normally called as the Null Geodesic method. The other method, appeared in [5], regards the action of the emitted particles across the classically forbidden region satisfies the relativistic Hamilton–Jacobi equation, and solving it yields the imaginary part of the action, which is an extension of the complex path analysis proposed by Padmanabhan et al. [6,7]. Till now, the tunnelling method has been already proved very robust for scalar particles across black hole horizons, and successfully recovered the correct Hawking temperatures of a wide variety of interesting and exotic spacetimes [8–19].

Recently, Kerner and Mann, modelling Hawking radiation as fermions tunnelling, have successfully recovered the Unruh and Hawking temperatures for the Rindler spacetime and general non-rotating black hole [20]. In the model, choosing a set of appropriate matrices γ^{μ} is an important technique, or we may not correctly recover the Hawking temperature we expected. And near the horizon, the imaginary part of the action is determined by the covariant Dirac equation. To further show the robustness of fermions tunnelling method, many recent papers appear to discuss Hawking radiation of Dirac particles via tunnelling from $(2+1)$-dimensional BTZ black hole [21], dynamical horizons [22], Kerr and Kerr–Newman black holes [23,24], charged dilatonic black holes [25] and rotating black holes in de Sitter spaces [26]. However these involved black holes share in taking 3- or 4-dimensional spacetimes. In this letter, by considering the Garfinkle–Horowitz–Strominger (GHS) [27] and charged non-extremal 5-dimensional D1–D5 black holes [28] in string theory, we once again confirm fermions tunnelling method. It is expected that our result strengthens the validity and power of the method.

The letter is outlined as follows. In Section 2, we begin with our studies by applying fermions tunnelling method to study Hawking radiation of Dirac particles across the GHS black hole. Section 3 is focus on fermions tunnelling from the charged non-extremal

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5-dimensional D1–D5 black hole. Section 4 ends up with some conclusions and discussions.

2. Fermions tunnelling from GHS black hole

In this section, we are devoted to study Hawking radiation of Dirac particles across the horizon of the GHS black hole in string theory by using fermions tunnelling method. The GHS black hole is a member of a family of solutions to low energy string theory described by the action

$$S = \int d^4x \sqrt{-g} e^{-2\phi}\left[-R - 4(\nabla\phi)^2 + F^2\right], \tag{1}$$

where ϕ is the dilaton field and $F_{\mu\nu}$ is the maxwell field associated with a $U(1)$ subgroup of $E_8 \times E_8$ or $\mathrm{Spin}(32)/Z_2$. The GHS black hole in the string frame is then given by

$$ds^2_{\text{string}} = -f(r)\,dt^2 + \frac{1}{h(r)}dr^2 + r^2\,d\Omega, \tag{2}$$

where

$$f(r) = \left(1 - \frac{2Me^{\phi_0}}{r}\right)\left(1 - \frac{Q^2 e^{3\phi_0}}{Mr}\right)^{-1},$$
$$h(r) = \left(1 - \frac{2Me^{\phi_0}}{r}\right)\left(1 - \frac{Q^2 e^{3\phi_0}}{Mr}\right).$$

Here ϕ_0 is the asymptotic constant value of the dilaton field. For $Q^2 < 2e^{-2\phi_0}M^2$, the metric (2) describes a black hole with an event horizon located at $r_h = 2Me^{\phi_0}$.

Now we focus on studying fermions tunnelling from the GHS black hole. The motion equation of Dirac fields Ψ in the curved spacetimes satisfies the following covariant Dirac equation as

$$i\gamma^\mu D_\mu \Psi + \frac{m}{\hbar}\Psi = 0, \tag{3}$$

where $D_\mu = \partial_\mu + \frac{i}{2}\Gamma^{\alpha\beta}_\mu \Sigma_{\alpha\beta}$ is the spin covariant derivative, $\Sigma_{\alpha\beta} = \frac{i}{4\pi}[\gamma^\alpha, \gamma^\beta]$ and m is the mass of the emitted particles. The matrices γ^μ is determined by $\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu} \times I$, where I is the identity matrix. In our case, we choose the matrices γ^μ for the GHS spacetime taking the form as

$$\gamma^t = \frac{1}{\sqrt{f(r)}}\begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix}, \qquad \gamma^r = \sqrt{h(r)}\begin{pmatrix} 0 & \sigma^3 \\ \sigma^3 & 0 \end{pmatrix},$$
$$\gamma^\theta = \frac{1}{r}\begin{pmatrix} 0 & \sigma^1 \\ \sigma^1 & 0 \end{pmatrix}, \qquad \gamma^\varphi = \frac{1}{r\sin\theta}\begin{pmatrix} 0 & \sigma^2 \\ \sigma^2 & 0 \end{pmatrix}, \tag{4}$$

and the σ^i $(i = 1, 2, 3)$ are the Pauli matrices, which are respectively described by

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \qquad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{5}$$

Dirac fields with the spin 1/2 have two spin states, that is the spin up and spin down states. When measuring the spin states along the r direction, the spin up state takes the same direction as r, but the spin down case has the opposite direction. In this letter, we only consider Dirac particles with the spin up without loss of generality, because after a manner fully analogous to the spin up case the same result will be present for Dirac particles with the spin down. Now we employ the following ansatz for Dirac particles with the spin up

$$\Psi_\uparrow(t, r, \theta, \varphi) = \begin{pmatrix} A(t, r, \theta, \varphi)\xi_\uparrow \\ B(t, r, \theta, \varphi)\xi_\uparrow \end{pmatrix} \exp\left[\frac{i}{\hbar} I_\uparrow(t, r, \theta, \varphi)\right]$$
$$= \begin{pmatrix} A(t, r, \theta, \varphi) \\ 0 \\ B(t, r, \theta, \varphi) \\ 0 \end{pmatrix} \exp\left[\frac{i}{\hbar} I_\uparrow(t, r, \theta, \varphi)\right], \tag{6}$$

where $\xi_\uparrow = \binom{1}{0}$ is the eigenvector of the matrix σ^3, and the corresponding eigenvalue is 1, which describes Dirac particles with the spin up. Applying the WKB approximation, and inserting the ansatz (6) into the covariant Dirac equation (3), and then dividing by the exponential term and multiplying by $\hbar$, the resulting equations to leading order in $\hbar$ take the forms as

$$B\left(\frac{1}{\sqrt{f(r)}}\partial_t I_\uparrow + \sqrt{h(r)}\,\partial_r I_\uparrow\right) - mA = 0, \tag{7}$$
$$B\left(\frac{1}{r}\partial_\theta I_\uparrow + \frac{i}{r\sin\theta}\partial_\varphi I_\uparrow\right) = 0, \tag{8}$$
$$A\left(\frac{1}{\sqrt{f(r)}}\partial_t I_\uparrow - \sqrt{h(r)}\,\partial_r I_\uparrow\right) + mB = 0, \tag{9}$$
$$A\left(\frac{1}{r}\partial_\theta I_\uparrow + \frac{i}{r\sin\theta}\partial_\varphi I_\uparrow\right) = 0. \tag{10}$$

Here the contributions of the derivatives A and B and the components $\Gamma^{\alpha\beta}_\mu \Sigma_{\alpha\beta}$ have already been neglected to the lowest order in WKB approximation due to the fact that they are all of order $\mathcal{O}(\hbar)$. To carry out the separation of variables for the above equations, considering the symmetries of the GHS spacetime we employ the ansatz

$$I_\uparrow = -\mathcal{E}t + \mathcal{J}(\theta, \phi) + \mathcal{W}(r), \tag{11}$$

where $\mathcal{E}$ is the energy of the emitted Dirac particle. Near the horizon, substituting the ansatz (11) into Eqs. (7), (8), (9) and (10) yields

$$B\left(-\frac{\mathcal{E}}{\sqrt{f_{,r}(r_h)(r - r_h)}} + \sqrt{h_{,r}(r_h)(r - r_h)}\,\partial_r \mathcal{W}(r)\right) - mA = 0, \tag{12}$$
$$B\left(\frac{1}{r}\partial_\theta \mathcal{J}(\theta, \phi) + \frac{i}{r\sin\theta}\partial_\phi \mathcal{J}(\theta, \phi)\right) = 0, \tag{13}$$
$$A\left(-\frac{\mathcal{E}}{\sqrt{f_{,r}(r_h)(r - r_h)}} - \sqrt{h_{,r}(r_h)(r - r_h)}\,\partial_r \mathcal{W}(r)\right) + mB = 0, \tag{14}$$
$$A\left(\frac{1}{r}\partial_\theta \mathcal{J}(\theta, \phi) + \frac{i}{r\sin\theta}\partial_\phi \mathcal{J}(\theta, \phi)\right) = 0, \tag{15}$$

where $f_{,r}$ and $h_{,r}$ denote a derivative with respect to r. Careful analysis on the above equations, we find $\mathcal{J}$ must be a complex function, which means it will yield a contribution to the imaginary part of the action. However further studying shows that the contribution of $\mathcal{J}$ is completely the same for both the outgoing and ingoing solutions, and therefore its total contribution to the tunnelling rate is cancelled out when dividing the outgoing probability by the ingoing probability. Then it is no need to solve the equations about the complex function $\mathcal{J}$. Now our attention should be focus on the radial function $\mathcal{W}$. From Eqs. (12) and (14), there will be a non-trivial solution for A and B if and only if the determinant of the coefficient matrix vanishes, which results

$$\partial_r \mathcal{W}(r) = \pm\frac{\sqrt{\mathcal{E}^2 + f_{,r}(r_h)(r - r_h)m^2}}{\sqrt{f_{,r}(r_h)h_{,r}(r_h)}(r - r_h)}. \tag{16}$$

Integrating the pole at the horizon of the GHS black hole as in Refs. [16,17], we have

$$\mathcal{W}_\pm = \pm i\pi \frac{\mathcal{E}}{\sqrt{f_{,r}(r_h)h_{,r}(r_h)}}. \tag{17}$$

In Eq. (17), the +/− sign corresponds to the outgoing/incoming solutions. The WKB approximation tells us the tunnelling probability is related to the imaginary part of the action as $P = \exp[-\frac{2}{\hbar}\operatorname{Im} I]$, where I is the action of particles across the black hole horizon. Set $\hbar$ to unity, and the overall tunnelling rate can be written as