Physics Letters B 683 (2010) 50–54

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

The exact behavior of electromagnetic Faraday rotation in crossing a gravitational sandwich wave in general relativity

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ARTICLE INFO

Article history:
Received 10 June 2009
Received in revised form 11 November 2009
Accepted 19 November 2009
Available online 9 December 2009
Editor: S. Dodelson

PACS:
04.20.-q
02.40.-k
04.30.Nk

Keywords:
General relativity
Colliding waves
Electromagnetic waves and gravitational sandwich waves
Faraday effect

ABSTRACT

We have analyzed the exact behavior of the polarization vector of a linearly polarized electromagnetic shock wave upon crossing a gravitational sandwich wave, by using Einstein's theory of general relativity. The Faraday rotation in the polarization vector of the electromagnetic field is induced in this nonlinear process. We show that the Faraday's angle highly depends on the electromagnetic parameter, gravitational parameter and the width of the gravitational sandwich wave.

1. Introduction

Colliding waves in General Relativity is a well-known subject by now [1] and it constitutes one of the nonlinear interaction effects of the Einstein's theory of relativity. As an example, we will consider the problem of colliding electromagnetic (em) wave with a gravitational shock wave. This problem was studied long time ago [2] and later on it was extended by introducing a second boundary on the gravitational wave (GW) so that it turns into a gravitational sandwich wave (GSW) [3]. Under this condition the em wave will be crossing a region with a GW pulse of finite duration. The geometry of GSW has a curved, bounded region in one null direction and are infinitely extended, otherwise. The origin of GSW may be attributed to the gravitational explosions emitting intermittent bursts of radiation.

The rotation of the plane of polarization of em waves has been studied extensively in the literature. For example in astrophysics [4–6] where detecting GWs through their interaction with light from distant sources is an interesting and important topic. Concerning the rotation of the plane of polarization of em waves it is worth to mention the studies done in [7]. In Ref. [7] they use the post-Minkowskian solution of the linearized Einstein field equations. It is shown that, the rotation angle of the plane of em waves is a boundary effect (geometric rotation) which vanishes for localized (astrophysical) GWs and is non-zero, but nevertheless negligible, for cosmological GWs. Also it is mentioned that the detection of GWs through the rotation of plane of polarization of light from distant sources is not feasible with technology currently available or foreseeable in the near future. However in [8] a prototype GW detector has been constructed for observations at 100 MHz where a resonance effect is used to improve the sensitivity by a factor of ~2000 over a narrow bandwidth. In Ref. [8] they also suggested possible technological pathways for improving the sensitivity.

Recall that Faraday rotation for GWs has been studied by some authors [9–11]. For example, Ruggiero and Tartaglia [12] have studied the gravitational Faraday rotation, on linearly polarized light rays emitted by a pulsar, orbiting another compact object. In their paper they have obtained a formula which relates the rotation angle to the orbital phase of the emitting pulsar, as well as to the parameters describing its orbit and to the orientation of the angular momentum of the binary companion.

Recently, Halilsoy and Gurtug [13] have considered the problem of collision of a linearly polarized shock em wave with both a cross polarized impulsive and shock GWs. As a result of this nonlinear

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0370-2693/$ – see front matter
doi:10.1016/j.physletb.2009.11.052

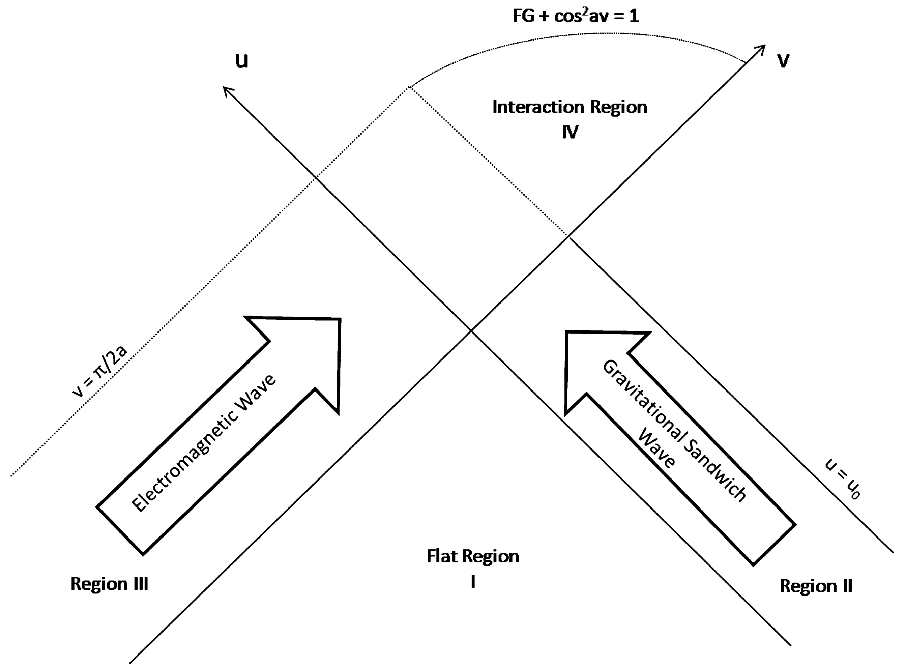

Fig. 1. The space–time diagram describes the collision of a plane of em wave propagating in one of the incoming regions and a plane GSW propagating in the other incoming region. The singular hypersurface occurs when $FG + \cos^2 av = 1$ in the interaction region.

process a detectable Faraday rotation in the polarization vector of the em field is induced which lead them to propose a method that renders indirect detection of strong GWs. This method is based on the reflection of a linearly polarized em shock wave from a cross polarized impulsive and shock GW. Therefore it is possible to probe controllable em waves in sensing the passing of a strong GW.

In this Letter we will consider the interaction of linearly polarized plane em wave with a cross polarized GSW. Again this highly nonlinear process induces a Faraday rotation in the polarization vector of the em field. We will analyze the exact behavior of the polarization vector of the em wave upon encountering with GSW. This is done by studying the effect of em parameters, gravitational parameter, and the width of the GSW on the Faraday's angle.

The Letter is organized as follows: In Section 2, we review the exact solution of electromagnetic wave crossing a GSW. In Section 3, we present the calculation of Faraday rotation. Next, we plot Faraday's angle and discuss their results. Finally, in Section 4, we draw our conclusions.

2. Electromagnetic wave crossing a gravitational sandwich wave

The interaction of a linearly polarized plane em wave with a cross polarized GSW that propagate in the opposite direction in each of the incoming regions is illustrated in Fig. 1.

The incoming region III ($v > 0$, $u < 0$) consists of a linearly polarized plane em wave described by the line element

$$ds^2 = 2\,du\,dv - H^2(v)\big(dx^2 + dy^2\big), \tag{1}$$

where

$$H(v) = \cos av\,\theta(v), \tag{2}$$

with $\theta(v)$ the unit step function and a is the energy (frequency) constant of the em wave described by the Ricci tensor component

$$\Phi_{22} = a^2\theta(v). \tag{3}$$

The incoming region II ($v < 0$, $u > 0$) contains a plane GSW described by

$$ds^2 = 2\,du\,dv - (F\,d\bar{x})^2 - (G\,d\bar{y})^2, \tag{4}$$

where

$$F(u) = \cosh b\widetilde{U} + b(\sinh bu_0)(u - u_0)\theta(u - u_0),$$
$$G(u) = \cos b\widetilde{U} - b(\sin bu_0)(u - u_0)\theta(u - u_0), \tag{5}$$

in which b represents the frequency of the GSW, and $\widetilde{U}$ is given as

$$\widetilde{U}(u) = u\theta(u) - (u - u_0)\theta(u - u_0), \tag{6}$$

where $u_0 > 0$ is the constant defining the width of the GSW. In an appropriate null tetrad the only nonvanishing Weyl curvature is

$$\Psi_4(u) = b^2\big[\theta(u - u_0) - \theta(u)\big]. \tag{7}$$

This shows that for $0 < u < u_0$ we have a constant curvature zone which vanishes elsewhere. It is seen that in region III the em field develops a fold singularity at $v = \pi/2a$. Thus, in the interaction region ($u > u_0$, $v > 0$), we are confined in a region $av < \pi/2$, $u > u_0$ to study the Faraday rotation. Before going to the interaction region let us transform the $(\bar{x}, \bar{y})$ axes to align them along with (x, y) by rotating the axes by an angle α. As a result a cross polarization term will rise in Eq. (4).

At, $u = 0 = v$ an incoming em wave from left encounters the GSW from right developing a space–time region described by line element given in Rosen form as

$$ds^2 = 2e^{-M}\,du\,dv - e^{-U}\big[\big(e^V\,dx^2 + e^{-V}\,dy^2\big)\cosh W - 2\sinh W\,dx\,dy\big]. \tag{8}$$

In the interaction region, we adapt the exact solution given by [2,3];

$$e^{-U} = FG - 1 + H^2,$$
$$e^{2V} = \frac{F^2\cos^2(\alpha/2) + G^2\sin^2(\alpha/2)}{F^2\sin^2(\alpha/2) + G^2\cos^2(\alpha/2)},$$
$$e^{-M} = H(FG)^{1/2}e^{U/2},$$
$$\sinh W = \frac{1}{2}\left(\frac{F^2 - G^2}{FG}\right)\sin\alpha,$$