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Operational definition of (brane-induced) space–time and constraints on the fundamental parameters ☆

Michael Maziashvili

Andronikashvili Institute of Physics, 6 Tamarashvili St., Tbilisi 0177, Georgia

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ABSTRACT

First we contemplate the operational definition of space–time in four dimensions in light of basic principles of quantum mechanics and general relativity and consider some of its phenomenological consequences. The quantum gravitational fluctuations of the background metric that comes through the operational definition of space–time are controlled by the Planck scale and are therefore strongly suppressed. Then we extend our analysis to the braneworld setup with low fundamental scale of gravity. It is observed that in this case the quantum gravitational fluctuations on the brane may become unacceptably large. The magnification of fluctuations is not linked directly to the low quantum gravity scale but rather to the higher-dimensional modification of Newton's inverse square law at relatively large distances. For models with compact extra dimensions the shape modulus of extra space can be used as a most natural and safe stabilization mechanism against these fluctuations.

1. Introduction

From the inception of quantum mechanics the physical quantities are usually understood to be observable, that is, they should be specified in terms of real or Gedanken measurements performed by well-prescribed measuring procedures. The concept of measurement has proved to be a fundamental notion for revealing the genuine nature of physical reality [1]. Space–time representing a frame in which everything takes place is one of the most fundamental concepts in physics. The importance of operational definition of physical quantities gives a strong motivation for a critical view how one actually measures the space–time geometry [2,3]. The first natural question in this way is to understand to what maximal precision can we mark a point in space by placing there a test particle. Throughout this Letter we will use system of units $\hbar = c = 1$. In the framework of quantum field theory a quantum takes up at least a volume, δx^3, defined by its Compton wavelength $\delta x \gtrsim 1/m$. Not to collapse into a black hole, general relativity insists the quantum on taking up a finite amount of room defined by its gravitational radius $\delta x \gtrsim l_P^2 m$. Combining together both quantum mechanical and general relativistic requirements one finds

$$\delta x \gtrsim \max\left(m^{-1}, l_P^2 m\right). \tag{1}$$

From this equation one sees that a quantum occupies at least the volume $\sim l_P^3$. Therefore in the operational sense the point cannot be marked to a better accuracy than $\sim l_P^3$. As any measurement we can perform (real or Gedanken) is based on the using of quanta, from Eq. (1) one infers that we can never probe a length to a better accuracy than $\sim l_P$. Since our understanding of time is tightly related to the periodic motion along some length scale, this result implies in general an impossibility of space–time distance measurement to a better accuracy than $\sim l_P$. This point of view was carefully elaborated in [3]. This apparently trivial conclusion encountered serious bias when it was originally suggested by Mead [4]. Starting from the 1980s the operational definition of space–time attracted considerable continuing interest [5–10].

Our fundamental theories of physics involve huge hierarchies between the energy scales characteristic of gravitation $E_P = 1/\sqrt{G_N} \sim 10^{28}$ eV and particle physics $E_{\rm EW} \sim 1$ TeV. In the atomic and subatomic world therefore, gravity is so weak as to be negligible. This is one reason gravity is not included as part of the Standard Model of particle physics. But when energy scale approaches the Planck one gravity enters the game. The question of operational definition of space–time becomes particularly interesting and important in regard with the higher-dimensional theories with low quantum scale of gravity (close to the electroweak scale). First we summarize different approaches for operational definition of Minkowskian space–time that enables one to estimate the rate of quantum-gravitational fluctuations of the background metric. Then we address some of the implications of these fluctuations. Having discussed the case of 4D space–time, we generalize the operational definition to the brane-induced space–time and consider its phenomenological consequences.

☆ Extended version of the talk given at ICTP.

E-mail address: mishamazia@hotmail.com.

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2. Károlyházy uncertainty relation

2.1. Approach 1

For space–time measurement an unanimously accepted method one can find in almost every textbook of general relativity consists in using clocks and light signals [11]. Let us consider a light-clock consisting of a spherical mirror inside which light is bouncing. That is, a light-clock counts the number of reflections of a pulse of light propagating inside a spherical mirror. Therefore the precision of such a clock is set by the radius of the clock $\sim r_c$. It is clear that physically the coordinate system is defined only by explicitly carrying out the space–time distance measurements. Let us consider the construction of a coordinate system for a time interval t and with a spatial fineness δx in a Minkowski space–time [10]. Since a clock must be localized in a region with the size δx, the clock inevitably has a momentum of the order $\delta p \sim 1/\delta x$, obtained from the uncertainty relation of quantum mechanics. Thus the clock moves with a finite velocity of order $\delta v \sim 1/m\delta x$, where m denotes the mass of the clock. This implies that the coordinate system will be destroyed by the quantum effect in a finite period $\delta x/\delta v \sim m(\delta x)^2$. This period must be larger than the time interval t of the coordinate. Hence we obtain

$$t \lesssim m(\delta x)^2. \tag{2}$$

This gives a lower bound for the clock mass m for given t and δx. From Eq. (2), we need clock with a larger mass to construct a finer coordinate system. However there is an upper bound on the clock mass for no clock should become a black hole. Thus the clock's Schwarzschild radius should not exceed the localization region of the clock:

$$l_P^2 m \lesssim \delta x. \tag{3}$$

The clock mass can be chosen arbitrary if it satisfies Eq. (2) and Eq. (3). Combining Eqs. (2), (3) one gets

$$l_P^2 t \lesssim (\delta x)^3. \tag{4}$$

Taking note that our light-clock having the size δx cannot measure the time to a better accuracy than $\delta t = \delta x$ one arrives at the equation

$$\delta t_{\min} \simeq t_P^{2/3} t^{1/3}. \tag{5}$$

Eq. (5) was first obtained by Károlyházy in 1966 and was subsequently analyzed by him and his collaborators in much detail [12]. Notice that throughout this discussion the clock parameters allowing maximum precision in measuring the length scale l (that is, the optimal clock parameters) are as follows

$$r_c \simeq l_P^{2/3} l^{1/3}, \qquad m \simeq \frac{l^{1/3}}{l_P^{4/3}}. \tag{6}$$

2.2. Approach 2

It is instructive to take into account gravitational time delay of the clock [13]. After introducing the clock the metric takes the form

$$ds^2 = \left(1 - \frac{2l_P^2 m}{r}\right)dt^2 - \left(1 - \frac{2l_P^2 m}{r}\right)^{-1} dr^2 - r^2\, d\Omega^2.$$

The time measured by this clock is related to the Minkowskian time as [11]

$$t' = \left(1 - \frac{2l_P^2 m}{r_c}\right)^{1/2} t.$$

From this expression one sees that the disturbance of the background metric to be small, the size of the clock should be much greater than its gravitational radius $r_c \gg 2l_p^2 m$. Under this assumption for gravitational disturbance in time measurement one finds

$$t' = \left(1 - \frac{l_P^2 m}{r_c}\right)t.$$

Since we are using light-clock its mass cannot be less than π/r_c, which by taking into account that the size of the clock determining its resolution time represents in itself an error during the time measurement gives

$$\delta t = 2r_c + \pi \frac{t t_P^2}{r_c^2},$$

which after minimization with respect to r_c leads to Eq. (5).

The final result in the above approaches is the same Eq. (5). Nevertheless the second approach strongly discourages to take the optimal size of the clock to be close to its gravitational radius (6). For the optimal parameters of the clock in measuring the space–time distance l one finds

$$r_c \simeq l_P^{2/3} l^{1/3}, \qquad m \simeq \frac{1}{r_c}.$$

3. Field theory view

Effective quantum field theory with built in IR and UV cutoffs satisfying the black-hole entropy bound leads to Eq. (5), where l and δl play the roles of IR and UV scales respectively [14]. For an effective quantum field theory in a box of size l with UV cutoff Λ the entropy S scales as

$$S \sim l^3 \Lambda^3.$$

That is, the effective quantum field theory counts the degrees of freedom simply as the numbers of cells Λ^{-3} in the box l^3. Nevertheless, considerations involving black holes demonstrate that the maximum entropy in a box of volume l^3 grows only as the area of the box [15]

$$S_{\mathrm{BH}} \simeq \left(\frac{l}{l_P}\right)^2.$$

So that, with respect to the Bekenstein bound [15] the degrees of freedom in the volume should be counted by the number of surface cells l_P^2. A consistent physical picture can be constructed by imposing a relationship between UV and IR cutoffs [14]

$$l^3 \Lambda^3 \lesssim S_{\mathrm{BH}} \simeq \left(\frac{l}{l_P}\right)^2. \tag{7}$$

Consequently, one arrives at the conclusion that the length l, which serves as an IR cutoff, cannot be chosen independently of the UV cutoff, and scales as Λ^{-3}. Rewriting this relation wholly in length terms, $\delta l \equiv \Lambda^{-1}$, one arrives at Eq. (5). Is it an accidental coincidence? Indeed not. The relation (7) can be simply understood from Eq. (5). The IR scale l cannot be given to a better accuracy than $\delta l \simeq l_P^{2/3} l^{1/3}$. Therefore, one cannot measure the volume l^3 to a better precision than $\delta l^3 \simeq l_P^2 l$ and correspondingly maximal number of cells inside the volume l^3 that may make an operational sense is given by $(l/l_P)^2$. Thus the Károlyházy relation implies the black-hole entropy bound given by Eq. (7). These ideas lead to the far reaching holographic principle for an ultimate unification that may perhaps be achieved when the basic aspects of quantum theory, particle theory and general relativity are combined [16].

4. Energy density of the fluctuations

Károlyházy uncertainty relation naturally translates into the metric fluctuations, as if it was possible to measure the metric precisely one could estimate the length between two points exactly.