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On Wess–Zumino gauge

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ABSTRACT

In the relation between the linear (L) supersymmetry (SUSY) representation and the nonlinear (NL) SUSY representation we discuss the role of the Wess–Zumino gauge. We show in two-dimensional spacetime that a spontaneously broken LSUSY theory with mass and Yukawa interaction terms for a minimal off-shell vector supermultiplet is obtained from a general superfield without imposing any special gauge conditions in $N=2$ NL/L SUSY relation.

Superfields on superspace provide a method to formulate supersymmetric (SUSY) field theories systematically [1,2]. The Wess–Zumino (WZ) gauge is used conveniently to eliminate redundant (gauge) degrees of freedom (d.o.f.) from general superfields for vector supermultiplets. Linear (L) SUSY actions for minimal off-shell vector supermultiplets (in interacting theories) are constructed by using superfields in the WZ gauge.

On the other hand, various LSUSY theories are related (equivalent) to a nonlinear (NL) SUSY model [3] through the linearization of NLSUSY both for the free case [4–8] and for the interacting (SUSY-QED) case [9,10], which are essential to study the low energy physics and cosmology of NLSUSY general relativity (GR) [11] in the SGM scenario [12]. In the NL/L SUSY relation, all component fields for the vector supermultiplets including the redundant ones of the general superfields are consistently expressed as composites of Nambu–Goldstone (NG) fermion [7,13] (called *superon* in the SGM scenario), whose expressions are called *SUSY invariant relations*. This means the WZ gauge condition used in LSUSY theories gives apparently the artificial (gauge dependent) restrictions on the composite states of NG fermions, which looks unnatural so far provided the composites are regarded as some eigen state of spacetime symmetry. Therefore, it is worthwhile studying the general properties of the superfield formulation which does not rely on the special gauge condition in viewpoints of NL/L SUSY relation.

In this Letter, we focus on the interacting SUSY theory for the $N=2$ vector supermultiplet in two-dimensional spacetime ($d=2$) for simplicity. Note that $N=1$ vector supermultiplet is unphysical [14] in the SGM scenario. We show that by using the $N=2$ NL/L SUSY relation constructed systematically in the previous paper [13] the interacting SUSY theory for the minimal off-shell vector supermultiplet and for the general vector supermultiplet as well accompanying the spontaneous SUSY breaking are obtained from the general superfield without imposing any special gauge conditions.

The $N=2$ general superfield on superspace coordinates (x^a, θ^i) for the $N=2$ LSUSY vector supermultiplet (in $d=2$) is given by [15,16]

$$\mathcal{V}(x,\theta^i) = C(x) + \bar\theta^i \Lambda^i(x) + \frac{1}{2}\bar\theta^i\theta^j M^{ij}(x) - \frac{1}{2}\bar\theta^i\theta^i M^{jj}(x) + \frac{1}{4}\epsilon^{ij}\bar\theta^i\gamma_5\theta^j\phi(x) - \frac{i}{4}\epsilon^{ij}\bar\theta^i\gamma_a\theta^j v^a(x) - \frac{1}{2}\bar\theta^i\theta^i\bar\theta^j\lambda^j(x) - \frac{1}{8}\bar\theta^i\theta^i\bar\theta^j\theta^j D(x), \quad (1)$$

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where $M^{ij} = M^{(ij)}$ $(= \frac{1}{2}(M^{ij} + M^{ji}))$ and $M^{ii} = \delta^{ij} M^{ij}$. LSUSY transformations for the (general) component fields $(C, \Lambda^i, M^{ij}, \ldots)$ with constant (Majorana) spinor parameters ζ^i are deduced from the following supertranslation of the superfield in the superspace (x^a, θ^i),

$$\delta_\zeta \mathcal{V}(x, \theta^i) = \bar{\zeta}^i Q^i \mathcal{V}(x, \theta^i) \tag{2}$$

with supercharges $Q^i = \frac{\partial}{\partial \bar{\theta}^i} + i \partial\!\!\!/ \theta^i$. Gauge transformations for $(C, \Lambda^i, M^{ij}, \ldots)$ are also defined as follows by means of [16]

$$\delta_g \mathcal{V}(x, \theta^i) = \Phi^1(x, \theta^i) + \Phi^2(x, \theta^i), \tag{3}$$

where Φ^i $(i = 1, 2)$ are two scalar superfield parameters for generalized gauge parameters (for a $N = 2$ matter scalar supermultiplet (for example, see [6])) (for simplicity, we put $\alpha = 1$ for the gauge transformations, $\delta_g \mathcal{V} = \Phi^1 + \alpha \Phi^2$, in Ref. [16]).

As in the case for $d = 4$, $N = 1$ SUSY [1], gauge invariant quantities for $N = 2$ denoted tentatively as $(A_0, \phi_0, F_{0ab}, \lambda_0, D_0)$ can be easily constructed by the explicit component form of the gauge transformation (3) as follows:

$$\left(A_0, \phi_0, F_{0ab}, \lambda_0^i, D_0\right) \equiv \left(M^{ii}, \phi, F_{ab}, \lambda^i + i \partial\!\!\!/ \Lambda^i, D + \Box C\right), \tag{4}$$

where $F_{0ab} = \partial_a v_{0b} - \partial_b v_{0a}$, $F_{ab} = \partial_a v_b - \partial_b v_a$ and $v_{0a} = v_a$ transforms as an Abelian gauge field. The familiar LSUSY transformations are calculated directly from Eq. (2). And for the minimal off-shell vector supermultiplet $(A_0, \phi_0, v_{0a}, \lambda_0^i, D_0)$ [9], we obtain

$$\begin{aligned}
&\delta_\zeta A_0 = \bar{\zeta}^i \lambda_0^i, \\
&\delta_\zeta \phi_0 = -\epsilon^{ij} \bar{\zeta}^i \gamma_5 \lambda_0^j, \\
&\delta_\zeta v_{0a} = -i \epsilon^{ij} \bar{\zeta}^i \gamma_a \lambda_0^j + \partial_a W\left(\Lambda^i\right), \\
&\delta_\zeta \lambda_0^i = (D_0 - i \partial\!\!\!/ A_0) \zeta^i - i \epsilon^{ij} \gamma_5 \partial\!\!\!/ \phi_0 \zeta^j + \frac{1}{2} \epsilon^{ab} \epsilon^{ij} F_{0ab} \gamma_5 \zeta^j, \\
&\delta_\zeta D_0 = -i \bar{\zeta}^i \partial\!\!\!/ \lambda_0^i,
\end{aligned} \tag{5}$$

where remarkably the redundant components in Eq. (1), $(C, \Lambda^i, M^{12}, M^{11} - M^{22})$, do not appear apparently in the minimal off-shell vector multiplet $\delta_\zeta(A_0, \phi_0, v_{0a}, \lambda_0^i, D_0)$ disregarding the gauge choice except for the $U(1)$ gauge parameter $W(\Lambda^i) = -2\epsilon^{ij} \bar{\zeta}^i \ \Lambda^j$ composed of the fermionic d.o.f. That is, the gauge invariant sector and the redundant sector are mixed by only the gauge transformations with the fermionic parameters. Therefore the redundant sector plays important roles for constructing SUSY QED.

The LSUSY transformations (5) satisfy the closed commutator algebra,

$$[\delta_{\zeta_1}, \delta_{\zeta_2}] = \delta_P\left(\Xi^a\right), \tag{6}$$

where $\delta_P(\Xi^a)$ means a translation with a parameter $\Xi^a = 2i \bar{\zeta}_1^i \gamma^a \zeta_2^i$. Interestingly, in contrast to the WZ gauge, the commutator algebra for v_{0a} does not contain the $U(1)$ gauge transformation term due to

$$\delta_{\zeta_1} W_{\zeta_2}\left(\Lambda^i\right) - \delta_{\zeta_2} W_{\zeta_1}\left(\Lambda^i\right) = -2\left(\epsilon^{ij} \bar{\zeta}_1^i \zeta_2^j A_0 + \bar{\zeta}_1^i \gamma_5 \zeta_2^i \phi_0 - i \bar{\zeta}_1^i \gamma^a \zeta_2^i v_{0a}\right) \tag{7}$$

and the consequent cancellations.

Now we see the above general arguments in the viewpoints of NL/L SUSY relation. Let us introduce the SUSY invariant relations between $(C, \Lambda^i, M^{ij}, \ldots)$ of $N = 2$ LSUSY and (Majorana) NG fermions ψ^i of $N = 2$ NLSUSY [13] in $d = 2$, in which each component field is expanded in terms of ψ^i with respect to a constant κ whose dimension is $(\text{mass})^{-1}$ ($\kappa^{-2} \sim \Lambda/G$ in NLSUSY GR [11]) as

$$\left(C, \Lambda^i, M^{ij}, \ldots\right) \sim \xi \kappa^{n-1} \left(\psi^i\right)^n |w| \quad (n = 4, 3, \ldots, 0), \tag{8}$$

where ξ is an arbitrary dimensionless constant, $(\psi^i)^4 = \bar{\psi}^i \psi^i \bar{\psi}^j \psi^j$, $(\psi^i)^3 = \psi^i \bar{\psi}^j \psi^j$ and $(\psi^i)^2 = \bar{\psi}^i \psi^j, \epsilon^{ij} \bar{\psi}^i \gamma_5 \psi^j, \epsilon^{ij} \bar{\psi}^i \gamma^a \psi^j$, which are very promising features for SGM scenario [12]. In Eq. (8), $|w|$ is the determinant introduced in [3] describing the dynamics of NG fermions of NLSUSY, i.e. for the $d = 2$, $N = 2$ ($N > 2$, as well) NLSUSY case,

$$|w| = \det\left(w^a{}_b\right) = \det\left(\delta^a{}_b + t^a{}_b\right), \quad t^a{}_b = -i\kappa^2 \bar{\psi}^i \gamma^a \partial_b \psi^i, \tag{9}$$

expanded in terms of $t^a{}_b$ or ψ^i as

$$\begin{aligned}
|w| &= 1 + t^a{}_a + \frac{1}{2!}\left(t^a{}_a t^b{}_b - t^a{}_b t^b{}_a\right) \\
&= 1 - i\kappa^2 \bar{\psi}^i \partial\!\!\!/ \psi^i - \frac{1}{2}\kappa^4 \left(\bar{\psi}^i \partial\!\!\!/ \psi^i \bar{\psi}^j \partial\!\!\!/ \psi^j - \bar{\psi}^i \gamma^a \partial_b \psi^i \bar{\psi}^j \gamma^b \partial_a \psi^j\right) \\
&= 1 - i\kappa^2 \bar{\psi}^i \partial\!\!\!/ \psi^i - \frac{1}{2}\kappa^4 \epsilon^{ab} \left(\bar{\psi}^i \psi^j \partial_a \bar{\psi}^i \gamma_5 \partial_b \psi^j + \bar{\psi}^i \gamma_5 \psi^j \partial_a \bar{\psi}^i \partial_b \psi^j\right).
\end{aligned} \tag{10}$$

Note that the NG fermion fields ψ^i transform nonlinearly as

$$\delta_\zeta \psi^i = \frac{1}{\kappa} \zeta^i - i\kappa \bar{\zeta}^j \gamma^a \psi^j \partial_a \psi^i, \tag{11}$$

which also satisfy the commutator algebra (6).