Physics Letters B 666 (2008) 100–105

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Wrapping at four loops in $\mathcal{N}=4$ SYM

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ARTICLE INFO

Article history:
Received 18 June 2008
Accepted 26 June 2008
Available online 3 July 2008
Editor: L. Alvarez-Gaumé

PACS:
11.15.-q
11.30.Pb
11.25.Tq

Keywords:
Super-Yang–Mills
Superspace
Anomalous dimensions
Integrability
Wrapping

ABSTRACT

We present the planar four-loop anomalous dimension of the composite operator $\mathrm{tr}(\phi[Z,\phi]Z)$ in the flavour $SU(2)$ sector of the $\mathcal{N}=4$ SYM theory. At this loop order wrapping interactions are present: they give rise to contributions proportional to $\zeta(5)$ increasing the level of transcendentality of the anomalous dimension. In a sequel of this Letter all the details of our calculation will be reported.

1. Introduction

After the advent of the AdS/CFT conjecture [1] there has been a renewed interest in $\mathcal{N}=4$ SYM theory, which represents one of the best playgrounds to test new ideas connected to non-perturbative results.

The first prediction that one would like to test is the matching of the spectra on the two sides of the conjecture. In fact we expect that the spectrum of the anomalous dimensions of gauge invariant operators of the planar $\mathcal{N}=4$ SYM theory matches the spectrum of strings on $\mathrm{AdS}_5 \times \mathrm{S}^5$.

Thus it is very important to have tools for the computation of anomalous dimensions. A big progress in this direction has been made in the last five years after the realization [2,3] that the planar one-loop dilatation operator of $\mathcal{N}=4$ SYM maps into the Hamiltonian of an integrable spin chain. The spin chain picture revealed itself very fruitful in understanding the integrability properties of higher orders in perturbation theory [4,5]. In addition it suggested to compute anomalous dimensions while finding solutions of associated Bethe equations [6]. The form of these equations has been recently refined with the introduction of the so-called dressing phase [7–10].

Now the hope of a direct comparison between the spectra on the two sides of the AdS/CFT correspondence is definitely more concrete. However a major obstacle in pursuing this program is due to the fact that the Bethe ansatz is asymptotic, i.e. it applies only to long operators. Indeed the spin chain Hamiltonian is long range: at a given perturbative order K in the coupling constant

$$g = \frac{\sqrt{\lambda}}{4\pi} \tag{1.1}$$

(where $\lambda = g_{\mathrm{YM}}^2 N$ is the 't Hooft coupling) the range of the interactions between adjacent sites grows with the perturbative order as $K+1$. For an operator of length L we should expect new effects when the range exceeds L. The asymptotic Bethe ansatz breaks down at orders $K \geqslant L$ since the interaction is no longer localized in some limited region along the state and asymptotic states cannot be defined. This spreading of the interaction manifests itself with the insurgence of a new type of contributions, the so-called *wrapping interactions*.

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0370-2693/$ – see front matter
doi:10.1016/j.physletb.2008.06.061

Several papers have addressed this issue. In [11] the properties of wrapping interactions have been worked out in terms of Feynman diagrams. In [12] it was proposed that the thermodynamic Bethe ansatz captures the wrapping interactions. This work has been extended in [13,14]. In [15] it was assumed that wrapping might be described by the Hubbard model. Finally by using restrictions from the BFKL equation [16–18], a quantitative proposal for the anomalous dimensions at four-loop order with wrapping effects was conjectured in [19].

The aim of this Letter is to shed light on the present situation: we perform an explicit field theoretical anomalous dimension computation for the length-four Konishi descendant $\operatorname{tr}(\phi[Z,\phi]Z)$ at four loops. This is the simplest case in which wrapping effects are present. Different conjectures on the value of this anomalous dimension were proposed in [10,15,19].

A complete four-loop Feynman graph calculation is terribly complicated, so we had to find a simplifying strategy in order to proceed. The diagrams are naturally divided in two classes: four-loop graphs with no wrapping and graphs with wrapping interactions. We obtain the contributions from the two classes as follows:

First we take advantage of the known form of the four-loop dilatation operator D_4 given in [20]: all the non-wrapping contributions can be obtained by subtracting from D_4 its range 5 part. The remaining terms contain all the contributions with range from 1 to 4, so they can be applied safely to our length four operator. In this way we avoid the explicit computation of this vast and difficult class of Feynman graphs. This will be done in Section 2.

We consider wrapping interactions in Section 3. Many diagrams need to be considered but $\mathcal{N}=1$ supergraph techniques allow us to drastically simplify the calculation. After D-algebra manipulations the diagrams are reduced to standard four-loop momentum integrals which we compute by means of uniqueness and the Gegenbauer polynomial x-space technique [21,22].

Finally in Section 4 we collect all the terms and compute the four-loop planar anomalous dimension of the length four Konishi descendant. Our result shows that previous conjectures do not reproduce the correct anomalous dimension. In particular we find that at this loop order wrapping interactions give rise to contributions proportional to $\zeta(5)$ increasing the level of transcendentality of the anomalous dimension. In the following we describe the various steps that allowed us to reach the final result, while the details of the calculation will be reported in a separate publication [23].

2. Subtraction of range five interactions

In this section we compute the contributions to the anomalous dimension due to four-loop non-wrapping graphs. To this end we consider the four-loop planar dilatation operator in the $SU(2)$ subsector containing all operators made of two out of the three complex scalars of $\mathcal{N}=4$ SYM, denoted by ϕ and Z. It is given by [20]

$$\begin{aligned}
D_4 = {} & -(560+4\beta)\{\,\} \\
& + (1072+12\beta+8\epsilon_{3a})\{1\} \\
& - (84+6\beta+4\epsilon_{3a})\{1,3\} - 4\{1,4\} - (302+4\beta+8\epsilon_{3a})\big(\{1,2\}+\{2,1\}\big) \\
& + (4\beta+4\epsilon_{3a}+2i\epsilon_{3c}-4i\epsilon_{3d})\{1,3,2\} + (4\beta+4\epsilon_{3a}-2i\epsilon_{3c}+4i\epsilon_{3d})\{2,1,3\} \\
& + (4-2i\epsilon_{3c})\big(\{1,2,4\}+\{1,4,3\}\big) + (4+2i\epsilon_{3c})\big(\{1,3,4\}+\{2,1,4\}\big) + (96+4\epsilon_{3a})\big(\{1,2,3\}+\{3,2,1\}\big) \\
& - (12+2\beta+4\epsilon_{3a})\{2,1,3,2\} + (18+4\epsilon_{3a})\big(\{1,3,2,4\}+\{2,1,4,3\}\big) - (8+2\epsilon_{3a}+2i\epsilon_{3b})\big(\{1,2,4,3\}+\{1,4,3,2\}\big) \\
& - (8+2\epsilon_{3a}-2i\epsilon_{3b})\big(\{2,1,3,4\}+\{3,2,1,4\}\big) - 10\big(\{1,2,3,4\}+\{4,3,2,1\}\big),
\end{aligned} \tag{2.1}$$

where ϵ_{3a}, ϵ_{3b}, ϵ_{3c}, ϵ_{3d} parameterize the free choice of the renormalization scheme, and $\beta=4\zeta(3)$ comes from the dressing phase. The permutation structures are defined as

$$\{a_1,\ldots,a_n\} = \sum_{r=0}^{L-1} \mathrm{P}_{a_1+r,a_1+r+1}\cdots\mathrm{P}_{a_n+r,a_n+r+1} \tag{2.2}$$

for the action on a cyclic state with L sites, where $\mathrm{P}_{a,a+1}$ permutes the flavours of the ath and $(a+1)$th site. Some rules for the manipulation of these structures can be found in [24].

In order to obtain the four-loop contributions we are interested in, we cannot use the expression (2.1) directly since it contains terms which describe the permutations among five neighbouring legs. Hence it can be applied only to a state in the asymptotic sense, i.e. the number of sites in the state has to be five or more. If we want to obtain the sum of all four-loop Feynman diagrams using D_4, we can correct it for the application on a length four state: the contributions from all the diagrams which describe the interactions of five neighbouring legs have to be replaced by the contributions from all four-loop wrapping interactions.

The flavour permutation structure of each Feynman diagram is completely determined by the scalar interactions. As will be explained in [23] the relevant flavour exchanges can be uniquely captured in terms of the four functions

$$\begin{aligned}
\chi(a,b,c,d) &= \{\,\} - 4\{1\} + \{a,b\} + \{a,c\} + \{a,d\} + \{b,c\} + \{b,d\} + \{c,d\} - \{a,b,c\} - \{a,b,d\} - \{a,c,d\} - \{b,c,d\} + \{a,b,c,d\}, \\
\chi(a,b,c) &= -\{\,\} + 3\{1\} - \{a,b\} - \{a,c\} - \{b,c\} + \{a,b,c\}, \\
\chi(a,b) &= \{\,\} - 2\{1\} + \{a,b\}, \\
\chi(1) &= -\{\,\} + \{1\},
\end{aligned} \tag{2.3}$$

where the number of arguments $a,b,c,d=1,\ldots,4$ is given by the number of four-vertices. The independent flavour-exchange functions for the range five interactions are found by replacing a,b,c,d with the corresponding arguments of the range five permutation structures found in (2.1).

We have considered the contributions of all four-loop range five Feynman diagrams. As one important result we find that those diagrams, in which the first or the fifth line interacts with the rest of the graph only via flavour-neutral gauge bosons, cancel against