Physics Letters B 678 (2009) 174–180

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Noncommutativity effects in FRW scalar field cosmology

Behrooz Malekolkalami, Mehrdad Farhoudi *

Department of Physics, Shahid Beheshti University, G.C., Evin, Tehran 19839, Iran

ARTICLE INFO

Article history:
Received 22 November 2008
Received in revised form 16 May 2009
Accepted 11 June 2009
Available online 13 June 2009
Editor: T. Yanagida

PACS:
04.20.-q
04.90.+e
04.20.Fy
98.80.Qc

Keywords:
Noncommutative phase space
Scalar field cosmology
Quantum cosmology

ABSTRACT

We study effects of noncommutativity on the phase space generated by a non-minimal scalar field which is conformally coupled to the background curvature in an isotropic and homogeneous FRW cosmology. These effects are considered in two cases, when the potential of scalar field has zero and nonzero constant values. The investigation is carried out by means of a comparative detailed analysis of mathematical features of the evolution of universe and the most probable universe wave functions in classically commutative and noncommutative frames and quantum counterparts. The influence of noncommutativity is explored by the two noncommutative parameters of space and momentum sectors with a relative focus on the role of the noncommutative parameter of momentum sector. The solutions are presented with some of their numerical diagrams, in the commutative and noncommutative scenarios, and their properties are compared. We find that impose of noncommutativity in the momentum sector causes more ability in tuning time solutions of variables in classical level, and has more probable states of universe in quantum level. We also demonstrate that special solutions in classical and allowed wave functions in quantum models impose bounds on the values of noncommutative parameters.

1. Introduction

Scalar fields are an integral part of modern models in particle physics [1], and recently play very important roles in cosmology and have become a powerful tool to build cosmological models as well. They have key role in some of these models as current models of early cosmological inflation [2], or, in the viability of scalar field models as favorite candidates for dark matter [3]. Scalar field cosmological models have extensively been studied in the literatures, see, e.g., Ref. [4] and references therein. In the simplest interactions, a scalar field is coupled to gravity. In many cosmological models, scalar fields present degrees of freedom and appear as dynamical variables of corresponding phase space, where this point can be regarded as relevance of noncommutativity and these models.

The proposal of noncommutativity concept between space–time coordinates was introduced first by Snyder [5], and about twenty years ago, a mathematical theory, nowadays known as noncommutative geometry (NCG), has begun to take shape [6] based on this concept. In the last decay, study and investigation of physical theories in the noncommutative (NC) frame, like string and M-theory [7,8], has caused a renewed interest on noncommutativity in the classical and quantum fields. In particular, a novel interest has been developed in considering the NC classical and quantum cosmology. In these studies, the influence of noncommutativity has been explored by the formulation of a version of NC cosmology in which a deformation of minisuperspace [9–12] or, of phase space [13] is required instead of space–time deformation. From qualitative point of view, noncommutativity in the configuration space leads to general effects, however, a non-trivial noncommutativity in momentum sector introduces distinct effects in what concern with the behavior of dynamical variables.

Our purpose in this work is to build a NC scenario for the Friedmann–Robertson–Walker (FRW) cosmology including matter field via a deformation achieved by the Moyal product [7] in classical and quantum level. We introduce effects of noncommutativity by two parameters, namely θ and β, which are the NC parameters corresponding to space and momentum sectors, respectively. Then, we will show that impose of noncommutativity in the momentum sector causes more ability in tuning time solutions of variables in classical level, and has more probable states of universe in quantum level.

The Letter is organized as follows. In Section 2, we specify a model and inspect it in the classical version within the commutative and NC frames. Section 3 considers the quantum version of

* Corresponding author.
E-mail addresses: b_malekolkalami@sbu.ac.ir (B. Malekolkalami), m-farhoudi@sbu.ac.ir (M. Farhoudi).

0370-2693
doi:10.1016/j.physletb.2009.06.023

this model by investigating universe wave functions and compares their properties in the commutative and NC frames. A brief conclusion is presented in the last section.

2. The classical model

We consider a classical model consist of a cosmological system that is presented by a four-dimensional action with a non-minimally coupled scalar field to gravity in a FRW universe. To specify the NC effects of the model, we first treat the commutative version, and then the NC one in the following subsections.

2.1. Commutative phase space

A general action for a non-minimally coupled scalar field can be described by

$$\mathcal{A}=\int\sqrt{-g}\left[f(\phi)R-\frac{1}{2}g^{\mu\nu}\phi_{,\mu}\phi_{,\nu}-V(\phi)\right]d^4x, \tag{1}$$

where g is the determinant of the metric $g_{\mu\nu}$, R is the Ricci scalar, $V(\phi)$ and $f(\phi)$ are potential and coupling functions of the scalar field, respectively. We assume a homogeneous scalar field, that is $\phi=\phi(t)$, and the following FRW metric of the minisuperspace

$$ds^2=-N^2(t)\,dt^2+a^2(t)\left(\frac{dr^2}{1-kr^2}+r^2\,d\Omega^2\right), \tag{2}$$

where $N(t)$ is a lapse function, $a(t)$ is a scale factor and k specifies geometry of the universe. Substituting the metric (2) in action (1), one obtains the Lagrangian density

$$\mathcal{L}=6af\left(kN-\frac{\dot a^2}{N}\right)-6\frac{a^2\dot a\dot f}{N}+a^3N\left(\frac{\dot\phi^2}{2N^2}-V\right), \tag{3}$$

where total time derivative terms have been neglected.

We restrict our considerations to the case of conformally coupled scalar field model [12,14]. The general reason for selecting such a scalar field is that it allows exact solutions in simple cases, as those discussed along this work, and it is rich enough to be useful as a probe for the significant modifications that NCG introduces in classical and quantum cosmologies. That is, we set $f(\phi)=1/(2\kappa)-\xi\phi^2/2$, where $\kappa=8\pi G/c^4$ and ξ is the non-minimal coupling parameter that represents a direct coupling between the scalar field and curvature, and has an arbitrary value. Obviously, the case $\xi=0$ is the minimally coupling situation, however, as mentioned, we consider the conformal coupling case, i.e. $\xi=1/6$, and employ the unites $\hbar=1=c$ and $\kappa=3$.

Based on these assumptions and rescaling the scalar field as

$\chi=a\phi/\sqrt{2}$

the Lagrangian (3) reads

$$\mathcal{L}=kNa-\frac{a\dot a^2}{N}+\frac{a\dot\chi^2}{N}-\frac{kN\chi^2}{a}-a^3NV. \tag{4}$$

Thus, the corresponding Hamiltonian is

$$\mathcal{H}=N\left(-\frac{p_a^2}{4a}+\frac{p_\chi^2}{4a}-ka+\frac{k\chi^2}{a}+a^3V\right), \tag{5}$$

where p_a and p_χ are the canonical conjugate momenta. For the conformal time gauge selection, namely $N=a$, one gets

$$\mathcal{H}=-\frac{p_a^2}{4}+\frac{p_\chi^2}{4}-ka^2+k\chi^2+a^4V. \tag{6}$$

Then, the Hamilton equations are

$$\begin{aligned}
\dot a&=\{a,\mathcal{H}\}=-\frac{1}{2}p_a,\\
\dot{p_a}&=\{p_a,\mathcal{H}\}=2ka-4a^3V+\sqrt{2}\chi a^2V',\\
\dot\chi&=\{\chi,\mathcal{H}\}=\frac{1}{2}p_\chi,\\
\dot{p_\chi}&=\{p_\chi,\mathcal{H}\}=-2k\chi-\sqrt{2}a^3V',
\end{aligned} \tag{7}$$

where the prime denotes derivative with respect to ϕ.

In this work, in order to proceed further, we simply treat two special cases for the potential function, namely when there is no potential and when there is a non-zero constant value potential, $V=V_0$.

In free potential case, solutions to Eqs. (7), corresponding to the values of index curvature, with the Hamiltonian constraint, $\mathcal{H}\approx 0$, are as follows

$$k=1:\begin{cases}a(t)=A_1\cos t+A_2\sin t\text{ and }\chi(t)=B_1\cos t+B_2\sin t,\\ \text{with constraint: }A_1^2+A_2^2=B_1^2+B_2^2,\end{cases} \tag{8}$$

$$k=-1:\begin{cases}a(t)=A_3e^t+A_4e^{-t}\text{ and }\chi(t)=B_3e^t+B_4e^{-t},\\ \text{with constraint: }A_3A_4=B_3B_4,\end{cases} \tag{9}$$

$$k=0:\begin{cases}a(t)=A_5t+A_6\text{ and }\chi(t)=B_5t+B_6,\\ \text{with constraint: }A_5^2=B_5^2,\end{cases} \tag{10}$$

where A_i's and B_i's are constants of integration.

We will compare these solutions with their NC analogues in the next section, where we will also discuss the case of non-zero constant potential along with its NC correspondent.

2.2. Noncommutative phase space

Noncommutativity in classical physics is described by the Moyal product law (shown by the $*$ notation in below) between two arbitrary functions of phase space variables, namely $\zeta^a=(x^i,p^j)$ for $i=1,\dots,l$ and $j=l+1,\dots,2l$, as [7]

$$(f*g)(\zeta)=\exp\left[\frac{1}{2}\alpha^{ab}\partial_a^{(1)}\partial_b^{(2)}\right]f(\zeta_1)g(\zeta_2)\Big|_{\zeta_1=\zeta_2=\zeta}, \tag{11}$$

such that

$$(\alpha_{ab})=\begin{pmatrix}\theta_{ij} & \delta_{ij}+\sigma_{ij}\\ -\delta_{ij}-\sigma_{ij} & \beta_{ij}\end{pmatrix}, \tag{12}$$

where $a,b=1,2,\dots,2l$, θ_{ij} and β_{ij} are assumed to be elements of real and antisymmetric matrices, σ_{ij} is a symmetric matrix (which can be written as a combination of θ_{ij} and β_{ij}), and dimension of the classical phase space is $2l$. The deformed or modified Poisson brackets are defined as

$$\{f,g\}_\alpha=f*g-g*f, \tag{13}$$

and also the modified Poisson brackets of variables are

$$\{x_i,x_j\}_\alpha=\theta_{ij},\qquad\{x_i,p_j\}_\alpha=\delta_{ij}+\sigma_{ij}\quad\text{and}\quad\{p_i,p_j\}_\alpha=\beta_{ij}. \tag{14}$$

The simplest way to study physical theories within the NCG is replacement of the Moyal product with the ordinary multiplication. Actually, where variables of classical phase space obey the usual Poisson brackets, i.e. $\{x_i,x_j\}=0=\{p_i,p_j\}$ and $\{x_i,p_j\}=\delta_{ij}$, one can consider the following suitable linear, non-canonical, transformation

$$x_i'=x_i-\frac{1}{2}\theta_{ij}p^j\quad\text{and}\quad p_i'=p_i+\frac{1}{2}\beta_{ij}x^j, \tag{15}$$