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Equivalence between supersymmetric self-dual and Maxwell–Chern–Simons models coupled to a matter spinor superfield

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ABSTRACT

We study the duality of the supersymmetric self-dual and Maxwell–Chern–Simons theories coupled to a fermionic matter superfield, using a master action. This approach evades the difficulties inherent to the quartic couplings that appear when matter is represented by a scalar superfield. The price is that the spinorial matter superfield represents a unusual supersymmetric multiplet, whose main physical properties we also discuss.

1. Introduction

Duality is an important phenomenon in quantum field theory allowing to relate two different theories. One example in $(2+1)$D [1,2] is the equivalence between the self-dual (SD) model, which does not possess gauge invariance, and the gauge-invariant Maxwell–Chern–Simons (MCS) model [3]. Different aspects of this equivalence were studied in the literature, see for example [4–11]. The most important results of these papers were to establish the mapping between a massive Thirring model and the Maxwell–Chern–Simons theory, and between the self-dual model and the Maxwell–Chern–Simons theory. The equivalence was also studied in the supersymmetric counterparts of the SD and MCS models, both in the free case [12] as well as in the presence of interactions with a scalar matter superfield [13]. However, as we will argue shortly, there remains some delicate intricacies which motivated us to reexamine the duality in the supersymmetric case.

In the present decade, a considerable interest has been devoted to the study of field theories in noncommutative spacetime and the possibility of Lorentz symmetry violation, mainly due to their relevance to quantum gravity. In this context, the duality in a noncommutative spacetime was considered in [14], and in the presence of Lorentz violation in [15].

The duality between the models can in principle be proved within two frameworks. The first of them is the gauge embedding method [10,13], whose essence consists in the extension of the self-dual model to a gauge theory by adding to its Lagrangian carefully chosen terms that vanish on-shell. The equivalence of the resulting gauge model and the starting SD theory can be seen by comparing their equations of motion, and can also be tested at the quantum level. The second framework is the master action method, used, for example, in [2,9], based on some primordial action (the master action) involving both the MCS and SD fields, coupled to some matter. Integration of this master action over the MCS field yields the SD action, whereas integration over the SD produces the MCS action, with appropriate couplings to the matter in both cases. Proceeding one step further, one can integrate over the remaining SD field in the first case, or over the MCS field in the second, finding the same effective self-interaction for the matter in both situations.

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When the SD field is coupled to a bosonic matter, one complication arises, in the sense that the model is actually equivalent to a "modified" MCS theory, with a field-dependent factor in front of the Maxwell term [10]. The source of this complication is essentially the appearance of quartic vertices involving the matter and the vector fields. When considering the duality in the supersymmetric case, the most natural matter supermultiplet is represented by a scalar superfield, which also couples to the vector (fermionic) superfields with a quartic vertex, so the same difficulty arises: the supersymmetric SD model is equivalent to a modified MCS theory [13]. The presence of the quartic vertices also precludes an extension of the proof of the duality for noncommutative theories (which, however, have been studied in the context of the Seiberg–Witten map, see, for example, [16,17]). One might wonder whether an interaction with a fermionic superfield, which does not induce a quartic vertex in the classical action, could make the study of the duality more transparent, and the aim of this work is to show that this is so, at least in the commutative case. The price to pay is that the fermionic matter superfield we have to introduce in such a study describes a non-minimal supersymmetric multiplet, involving four bosonic and four fermionic degrees of freedom.

The structure of this Letter looks as follows. In Section 2, we present the master action, and use the equations of motion to establish the duality at the classical level. In Section 3, we study the duality at the quantum level, by inspecting the generating of the SMCS and SSD theories. All this work is made for quite general couplings; some particular cases are discussed in Section 4. In Section 5, the physical content of the fermionic matter superfield introduced by us is made explicit. In the Summary, the results are discussed; in particular, we comment on the possible extension of our work to the noncommutative spacetime.

2. The duality at the classical level

As a first step, we introduce the following master Lagrangian describing the interaction of a spinorial matter superfield Ψ^α with the spinor superfields f_α (which will be further identified with the self-dual superfield) and A_α (which will be further identified with the Maxwell–Chern–Simons superfield),

$$\mathcal{L}_{\text{master}} = -\frac{m^2}{2} f^\alpha f_\alpha + m f^\alpha W_\alpha + \frac{m}{2} A^\alpha W_\alpha + k^\alpha f_\alpha + j^\alpha A_\alpha + \mathcal{L}_M(\Psi), \tag{1}$$

where $\mathcal{L}_M(\Psi)$ is the quadratic Lagrangian for the spinor matter superfield Ψ^α; j^α and k^α are currents depending on this superfield. Explicit forms for $\mathcal{L}_M(\Psi)$ and the currents will be presented later, at the moment, we can say that j^α is necessarily conserved ($D_\alpha j^\alpha = 0$) due to gauge invariance. Here $W_\alpha \equiv \frac{1}{2} D^\beta D_\alpha A_\beta$ is the gauge-invariant superfield strength constructed from the superfield A_α. The Lagrangian $\mathcal{L}_{\text{master}}$ is the natural superfield generalization of the one used in [9], with the notations and conventions of [18].

The equations of motion for the A^α and f^α superfields derived from Eq. (1) can be used to obtain the duality at the classical level. Varying the action $\int d^5 z \, \mathcal{L}_{\text{master}}$ with respect to f^α we obtain

$$f_\alpha = \frac{1}{m^2} k_\alpha + \frac{1}{m} W_\alpha, \tag{2}$$

which, inserted in Eq. (1), yields $\mathcal{L}_{\text{master}} = \mathcal{L}_{\text{SMCS}}$, with

$$\mathcal{L}_{\text{SMCS}} = \frac{1}{2} W^\alpha W_\alpha + \frac{m}{2} A^\alpha W_\alpha - \frac{\alpha}{4} \left(D^\alpha A_\alpha\right)^2 + \left(j^\alpha + \frac{1}{2m} D^\beta D^\alpha k_\beta\right) A_\alpha + \frac{1}{2m^2} k^\alpha k_\alpha + \mathcal{L}_M(\Psi). \tag{3}$$

This last Lagrangian describes the supersymmetric Maxwell–Chern–Simons (SMCS) field coupled to the matter through the "minimal" coupling $A^\alpha j_\alpha$, plus a "magnetic" coupling $\frac{1}{2m} A^\alpha D^\beta D_\alpha k_\beta = \frac{1}{m} W^\alpha k_\alpha$, and a Thirring-like self-interaction $\frac{1}{2m^2} k^\alpha k_\alpha$ of the spinorial matter superfield.

Varying the master action with respect to A^α provides us with

$$W_\alpha + \Omega_\alpha + j_\alpha = 0, \tag{4}$$

where $\Omega^\alpha \equiv (1/2) D^\beta D^\alpha f_\beta$. At this point, we recall the projectors on the transversal and longitudinal parts of a fermionic superfield η^α,

$$\eta_\parallel^\alpha = -D^\alpha D^\beta \frac{1}{2D^2} \eta_\beta, \qquad \eta_\perp^\alpha = D^\beta D^\alpha \frac{1}{2D^2} \eta_\beta, \tag{5}$$

so that $D^\alpha \eta_\alpha^\perp = 0$. The explicit form of the transversal projector in Eq. (5) allows us to rewrite Eq. (4) as

$$A_\alpha^\perp = -f_\alpha^\perp - \frac{1}{mD^2} j_\alpha. \tag{6}$$

Substituting Eqs. (6) and (4) into the master Lagrangian, and taking into account that, if η^α is transversal, $\eta^\alpha \xi_\alpha = \eta^\alpha \xi_\alpha^\perp$ for any ξ_α, we obtain $\mathcal{L}_{\text{master}} = \mathcal{L}_{\text{SSD}}$, with

$$\mathcal{L}_{\text{SSD}} = -\frac{m}{2} f^\alpha \Omega_\alpha - \frac{m^2}{2} f^\alpha f_\alpha + \left(k^\alpha - j^\alpha\right) f_\alpha - \frac{1}{2} j^\alpha \frac{1}{mD^2} j_\alpha + \mathcal{L}_M(\Psi). \tag{7}$$

This Lagrangian describes the dynamics of a supersymmetric self-dual (SSD) superfield which, besides of the "minimal" coupling to the current k_α, is also coupled in a non-local way to the current j_α. Moreover, a non-local Thirring-like term for the j_α shows up.

Classically, the Lagrangians in Eqs. (3) and (7) are equivalent, thus establishing the duality between these SMCS and SSD models at the level of equations of motion. Indeed, we can find an explicit mapping between the superfields and currents of the SMCS theory to their counterparts in the SSD model, such that the corresponding equations of motion are mapped one to the other. The equations of motion derived from the SSD Lagrangian in Eq. (7) can be cast as

$$m\Omega_\alpha + m^2 f_\alpha + j^\alpha - k^\alpha = 0, \tag{8}$$

and

$$\frac{\delta}{\delta \Psi^\beta} \int d^5 z \, \mathcal{L}_M + \frac{\partial j^\alpha}{\partial \Psi^\beta} \left(-f_\alpha^\perp - \frac{1}{mD^2} j_\alpha\right) + \frac{\partial k^\alpha}{\partial \Psi^\beta} f_\alpha = 0. \tag{9}$$

Using the projection operators in Eq. (5), we split Eq. (8) in the longitudinal,

$$m^2 f_\alpha^\parallel = k_\alpha^\parallel, \tag{10}$$

and transversal parts,

$$m\Omega_\alpha^\perp + m^2 f_\alpha^\perp + j_\alpha - k_\alpha^\perp = 0. \tag{11}$$

Hereafter, we omit the $\perp$ in the current j since we know it is always transversal. We see that the longitudinal part of f is not dynamical, but algebraically related to the longitudinal part of k_α.

The equations of motion derived from the SMCS Lagrangian in Eq. (3) read,

$$\frac{1}{2} D^\beta D_\alpha W_\beta + m W_\alpha + j_\alpha + \frac{D^2}{m} k_\alpha^\perp = 0, \tag{12}$$