Available online at www.sciencedirect.com

ScienceDirect

Physics Letters B 663 (2008) 66–72

PHYSICS LETTERS B

www.elsevier.com/locate/physletb

Heavy-quark contributions to the ratio F_L/F_2 at low x

Alexey Yu. Illarionov [a], Bernd A. Kniehl [b,*], Anatoly V. Kotikov [b,1]

[a] *Scuola Internazionale Superiore di Studi Avanzati, Via Beirut, 2–4, 34014 Trieste, Italy*
[b] *II. Institut für Theoretische Physik, Universität Hamburg, Luruper Chaussee 149, 22761 Hamburg, Germany*

Received 9 January 2008; received in revised form 6 February 2008; accepted 7 February 2008

Available online 18 March 2008

Editor: A. Ringwald

Abstract

We study the heavy-quark contribution to the proton structure functions $F_2^i(x, Q^2)$ and $F_L^i(x, Q^2)$, with $i = c, b$, for small values of Bjorken's x variable at next-to-lading order and provide compact formulas for their ratios $R_i = F_L^i/F_2^i$ that are useful to extract $F_2^i(x, Q^2)$ from measurements of the doubly differential cross section of inclusive deep-inelastic scattering at DESY HERA. Our approach naturally explains why R_i is approximately independent of x and the details of the parton distributions in the low-x regime.

PACS: 12.38.-t; 12.38.Bx; 13.66.Bc; 13.85.Lg

Keywords: Quantum chromodynamics; Deep-inelastic scattering; Structure functions; Differential cross sections

1. Introduction

The totally inclusive cross section of deep-inelastic lepton–proton scattering (DIS) depends on the square s of the centre-of-mass energy, Bjorken's variable $x = Q^2/(2pq)$, and the inelasticity variable $y = Q^2/(xs)$, where p and q are the four-momenta of the proton and the virtual photon, respectively, and $Q^2 = -q^2 > 0$. The doubly differential cross section is parameterized in terms of the structure function F_2 and the longitudinal structure function F_L, as

$$\frac{d^2\sigma}{dx\,dy} = \frac{2\pi\alpha^2}{xQ^4}\big\{\big[1 + (1-y)^2\big]F_2\big(x, Q^2\big) - y^2 F_L\big(x, Q^2\big)\big\}, \tag{1}$$

where α is Sommerfeld's fine-structure constant. At small values of x, F_L becomes non-negligible and its contribution should be properly taken into account when F_2 is extracted from the measured cross section. The same is true also for the contributions F_2^i and F_L^i of F_2 and F_L due to the heavy quarks $i = c, b$.

Recently, the H1 [1–3] and ZEUS [4–6] Collaborations at HERA presented new data on F_2^c and F_2^b. At small x values, of order 10^{-4}, F_2^c was found to be around 25% of F_2, which is considerably larger than what was observed by the European Muon Collaboration (EMC) at CERN [7] at larger x values, where it was only around 1% of F_2. Extensive theoretical analyses in recent years have generally served to establish that the F_2^c data can be described through the perturbative generation of charm within QCD (see, for example, the review in Ref. [8] and references cited therein).

In the framework of Dokshitzer–Gribov–Lipatov–Altarelli–Parisi (DGLAP) dynamics [9], there are two basic methods to study heavy-flavour physics. One of them [10] is based on the massless evolution of parton distributions and the other one on the photon–

* Corresponding author.
E-mail addresses: illario@sissa.it (A.Yu. Illarionov), kniehl@desy.de (B.A. Kniehl), kotikov@theor.jinr.ru (A.V. Kotikov).
[1] On leave of absence from the Bogoliubov Laboratory of Theoretical Physics, Joint Institute for Nuclear Research, 141980 Dubna, Moscow region, Russia.

0370-2693/$ – see front matter
doi:10.1016/j.physletb.2008.02.068

gluon fusion (PGF) process [11]. There are also some interpolating schemes (see Ref. [12] and references cited therein). The present HERA data on F_2^c [2,3,5,6] are in good agreement with the modern theoretical predictions.

In earlier HERA analyses [1,4], F_L^c and F_L^b were taken to be zero for simplicity. Four years ago, the situation changed: in the ZEUS paper [5], the F_L^c contribution at next-to-leading order (NLO) was subtracted from the data; in Refs. [2,3], the H1 Collaboration introduced the reduced cross sections

$$\tilde{\sigma}^{i\bar{i}} = \frac{xQ^4}{2\pi\alpha^2[1+(1-y)^2]}\frac{d^2\sigma^{i\bar{i}}}{dx\,dy} = F_2^i(x,Q^2) - \frac{y^2}{1+(1-y)^2}F_L^i(x,Q^2) \tag{2}$$

for $i = c, b$ and thus extracted F_2^i at NLO by fitting their data. Very recently, a similar analysis, but for the doubly differential cross section $d^2\sigma^{i\bar{i}}/(dx\,dy)$ itself, has been performed by the ZEUS Collaboration [6].

In this Letter, we present compact low-x approximation formulae for the ratio $R_i = F_L^i/F_2^i$ at leading order (LO) and NLO, which greatly simplify the extraction of F_2^i from measurements of $d^2\sigma^{i\bar{i}}/(dx\,dy)$.

Low-x approximations are a topic of old vintage: Ralston's pocket partonometer represents a fast analytic algorithm producing asymptotic low-x estimates for the parton distribution functions (PDFs) of the sea-quarks and the gluon [13]; approximate low-x relations between the gluon PDF and F_L, without heavy-quark contributions, were elaborated at LO [14] and NLO [15]. In the large-Q^2 region, F_L^i is known at next-to-next-to-leading order [16].

2. Master formula

We now derive our master formula for $R_i(x,Q^2)$ appropriate for small values of x, which has the advantage of being independent of the PDFs $f_a(x,Q^2)$, with parton label $a = g, q, \bar{q}$, where q generically denotes the light-quark flavours. In the low-x range, where only the gluon and quark-singlet contributions matter, while the non-singlet contributions are negligibly small, we have[2]

$$F_k^i(x,Q^2) = \sum_{a=g,q,\bar{q}}\sum_{l=+,-} C_{k,a}^l(x,Q^2)\otimes x f_a^l(x,Q^2), \tag{3}$$

where $l = \pm$ labels the usual $+$ and $-$ linear combinations of the gluon and quark-singlet contributions, $C_{k,a}^l(x,Q^2)$ are the DIS coefficient functions, which can be calculated perturbatively in the parton model of QCD, μ is the renormalization scale appearing in the strong-coupling constant $\alpha_s(\mu)$, and the symbol $\otimes$ denotes convolution according to the usual prescription, $f(x)\otimes g(x) = \int_x^1 (dy/y) f(y) g(x/y)$. Massive kinematics requires that $C_{k,a}^l = 0$ for $x > b_i = 1/(1+4a_i)$, where $a_i = m_i^2/Q^2$. We take m_i to be the solution of $\overline{m}_i(m_i) = m_i$, where $\overline{m}_i(\mu)$ is defined in the modified minimal-subtraction ($\overline{\mathrm{MS}}$) scheme.

Exploiting the low-x asymptotic behaviour of $f_a^l(x,Q^2)$ [17,18],

$$f_a^l(x,Q^2) \overset{x\to 0}{\to} \frac{1}{x^{1+\delta_l}}\tilde{f}_a^l(x,Q^2), \tag{4}$$

where the rise of $\tilde{f}_a^l(x,Q^2)$ as $x\to 0$ is less than any power of x, Eq. (3) can be rewritten as

$$F_k^i(x,Q^2) \approx \sum_{a=g,q,\bar{q}}\sum_{l=+,-} M_{k,a}^l(1+\delta_l,Q^2) x f_a^l(x,Q^2), \tag{5}$$

where

$$M_{k,a}^l(n,Q^2) = \int_0^{b_i} dx\, x^{n-2} C_{k,a}^l(x,Q^2) \tag{6}$$

is the Mellin transform, which is to be analytically continued from integer values n to real values $1+\delta_l$.

As demonstrated in Refs. [19,20], HERA data support the modified Bessel-like behavior of PDFs at small x values predicted in the framework of the so-called generalized double-asymptotic scaling regime. In this approach, one has $M_{k,a}^+(1,Q^2) = M_{k,a}^-(1,Q^2)$ if $M_{k,a}^l(n,Q^2)$ are devoid of singularities in the limit $\delta_l \to 0$, as we assume for the time being. Such singularities actually occur at NLO, leading to modifications to be discussed in Section 4. Defining $M_{k,a}(1,Q^2) = M_{k,a}^{\pm}(1,Q^2)$ and using $f_a(x,Q^2) = \sum_{l=\pm} f_a^l(x,Q^2)$, Eq. (5) may be simplified to become

$$F_k^i(x,Q^2) \approx \sum_{a=g,q,\bar{q}} M_{k,a}(1,Q^2) x f_a(x,Q^2). \tag{7}$$

[2] Here and in the following, we suppress the variables μ and m_i in the argument lists of the structure and coefficient functions for the ease of notation.