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Towards NNLO accuracy in the QCD sum rule for the kaon distribution amplitude

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Abstract

We calculate the $O(\alpha_s)$ and $O(\alpha_s^2)$ gluon radiative corrections to the QCD sum rule for the first Gegenbauer moment a_1^K of the kaon light-cone distribution amplitude. The NNLO accuracy is achieved for the perturbative term and quark-condensate contributions to the sum rule. A complete factorization is implemented, removing logarithms of s-quark mass from the coefficients in the operator-product expansion. The sum rule with radiative corrections yields $a_1^K(1\ \text{GeV}) = 0.10 \pm 0.04$.

1. Light-cone distribution amplitudes (DA's) of hadrons enter various factorization formulae used for description of exclusive processes in QCD. The concept of DA's allows to describe collinear partons in an energetic hadron, separating long-distance dynamics from the perturbatively calculable hard-scattering amplitudes.

The set of DA's with a growing twist is especially useful for the pion and kaon because their intrinsically small masses make the collinear description more efficient. The lowest twist-2 DA has a transparent physical interpretation, describing the longitudinal momentum distribution in the quark–antiquark Fock-state of a meson. Switching from the pion to kaon, one encounters the $SU(3)_{fl}$-symmetry violation effects, which originate from the quark mass difference $m_s - m_{u,d}$. These effects have to be accounted as accurate as possible, in order to assess the $SU(3)_{fl}$ symmetry relations between the hadronic amplitudes with pions and kaons. Important examples are the relations between $B \to \pi\pi$ and $B \to \pi K, K\bar{K}$ charmless decay amplitudes employed in the studies of CP-violation and quark–flavour mixing.

The most essential $SU(3)_{fl}$-violating effects in the kaon twist-2 DA include the ratio of the decay constants f_K/f_π and the difference between the longitudinal momenta of strange and nonstrange quark–partons. This difference is proportional to the first moment a_1^K in the decomposition of the kaon twist-2 DA in Gegenbauer polynomials, whereas a_1^π vanishes in the isospin (G-parity) symmetry limit. In addition, the ratio of the second Gegenbauer moments a_2^K/a_2^π can also deviate from unity; the effects related to a_n^K at $n \geqslant 3$ are usually neglected.

In this Letter we concentrate on the determination of the asymmetry parameter $a_1^K(\mu)$ for the kaon, at a low scale $\mu \sim 1$ GeV. The method originally suggested in [1] and based on QCD sum rules [2] is employed. The most recent sum rule estimates of a_1^K were obtained in [3] and [4], where, in addition to the known leading-order (LO) results, the next-to-leading (NLO), $O(\alpha_s)$ correction to the quark-condensate contribution are taken into account. These calculations, together with the estimates [5,6] based on the operator identities, yield the interval (quoted as a best estimate in [7]): $a_1^K(1\ \text{GeV}) = 0.06 \pm 0.03$. The positive sign of a_1^K corresponds, as expected, to a larger average momentum of the heavier valence s-quark in the kaon.

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The aim of this work is to upgrade the precision of the QCD sum rule for a_1^K. We calculate the gluon radiative corrections to the perturbative and quark-condensate contributions in NNLO, including $O(\alpha_s)$ and $O(\alpha_s^2)$ corrections. This task is technically feasible, due to the currently achieved state-of-the-art in the calculations of multiloop effects in the two-point correlation functions with strange and nonstrange quarks. For the correlation function with scalar and pseudoscalar currents the $O(\alpha_s^4)$, five-loop accuracy has recently been achieved [8] and used, e.g., for the QCD sum rule determination of the strange quark mass [9–11]. In this case, in the perturbative expansion the $O(\alpha_s^2)$ terms are important numerically, which is one motivation to include these terms also in the sum rule for a_1^K. The correlation functions underlying the sum rules for Gegenbauer coefficients are however different, because the currents contain derivatives. Therefore, the calculation reported in the present Letter, involves a certain technical novelty. In addition, we clarify and take into account the mixing of operators that is necessary for the complete factorization of small and large scales in the correlation function. Our result for the first Gegenbauer moment of the kaon DA is:

$$a_1^K(1\ \text{GeV}) = 0.10 \pm 0.04. \tag{1}$$

In what follows, we introduce the correlation function, present the expressions for the new radiative corrections, derive the resulting QCD sum rule for a_1^K, including the new $O(\alpha_s)$ and $O(\alpha_s^2)$ terms and perform the numerical analysis.

2. The twist-2 DA of the kaon enters the standard expression for the light-cone expansion of the vacuum–kaon bilocal matrix element (we take K^- for definiteness):

$$\langle K^-(q)|\bar{s}(z)\gamma_\mu\gamma_5[z,-z]u(-z)|0\rangle_{z^2\to 0} = -iq_\mu f_K \int_0^1 du\, e^{iuq\cdot z - i\bar{u}q\cdot z}\varphi_K(u,\mu), \tag{2}$$

where the s- and $\bar{u}$-quarks carry the momentum fractions u and $\bar{u} = 1-u$; $[z,-z]$ is the path-ordered gauge-factor $[x_1,x_2] = P\exp(i\int_0^1 dv\,(x_1-x_2)_\rho A^\rho(vx_1+\bar{v}x_2))$, and μ is the normalization scale determined by the interval z^2 near the light-cone. We use the compact notation $A_\rho = g_s A_\rho^a \lambda^a/2$ for the gluon field and the covariant derivative is defined as $D_\rho = \partial_\rho - iA_\rho$. In (2), the twist-2 DA $\varphi_K(u)$ is normalized to unity, so that in the local limit $z \to 0$ one reproduces the definition of the kaon decay constant f_K.

As usual, $\varphi_K(u)$ is expanded in the Gegenbauer polynomials

$$\varphi_K(u,\mu) = 6u\bar{u}\left(1 + \sum_{n=1}^{\infty} a_n^K(\mu) C_n^{3/2}(u-\bar{u})\right), \tag{3}$$

with the coefficients $a_n^K(\mu)$ (Gegenbauer moments). The first Gegenbauer moment a_1^K is proportional to the average difference between the longitudinal momenta of the strange and nonstrange quarks in the two-parton state of the kaon. Expanding both parts of Eq. (2) around $z=0$ in local operators and using the decomposition (3) with $C_1^{3/2}(x) = 3x$ one relates a_1^K to the vacuum-to-kaon matrix element of a local operator with one derivative:

$$\langle K^-(q)|\bar{s}\gamma_\nu\gamma_5 i\overleftrightarrow{D}_\lambda u|0\rangle = -iq_\nu q_\lambda f_K \frac{3}{5} a_1^K, \tag{4}$$

where $\overleftrightarrow{D}_\lambda = \overrightarrow{D}_\lambda - \overleftarrow{D}_\lambda$.

The Gegenbauer moments $a_n^{\pi,K}(\mu)$ are known to be multiplicatively renormalizable only at the one-loop level. Generally, this property is lost at higher orders in α_s, e.g., the two-loop renormalization of a_2^π calculated in [12] includes operator-mixing effects. Still the a_1^K case is special, in so far as the underlying operator $\bar{s}\gamma_\nu\gamma_5 i\overleftrightarrow{D}_\lambda u$ can only mix with $\partial_\lambda(\bar{s}\gamma_\nu\gamma_5 u)$, as there is no other local operator with the same dimension and flavour content. However, the above two operators have opposite $G_{(s)}$-parities, where $G_{(s)}$ is the analog of the isospin G-parity for the $SU(2)$ subgroups of $SU(3)_{fl}$ involving s quark (V- or U-spins). Naturally, $G_{(s)}$-conservation is only realized in the $m_s = m_{u,d}$ limit. Note, however, that the ultraviolet renormalization in $\overline{\text{MS}}$-scheme is a mass-independent procedure. Hence it is legitimate to consider the $SU(3)_{fl}$ limit, while performing the renormalization, so that the $G_{(s)}$-conservation protects the operators from mixing with each other. As a result, a_1^K remains multiplicatively renormalizable at any order in perturbation theory. For completeness, we present the well-known expression for scale-dependence of a_1^K with the two-loop (NLO) accuracy, written in an unexpanded form:

$$a_1^K(\mu) = \left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right)^{\frac{\gamma_0}{\beta_0}} \left(\frac{\beta_0 + \beta_1(\alpha_s(\mu_0)/\pi)}{\beta_0 + \beta_1(\alpha_s(\mu)/\pi)}\right)^{(\frac{\gamma_0}{\beta_0} - \frac{\gamma_1}{\beta_1})} a_1^K(\mu_0), \tag{5}$$

where $\gamma_0 = 8/9$, $\gamma_1 = 590/243$ are the anomalous dimensions [13] and $\beta_0 = 9/4$, $\beta_1 = 4$ are the coefficients of β-function for $n_f = 3$.