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Covariant anomalies and Hawking radiation from charged rotating black strings in anti-de Sitter spacetimes

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Abstract

Motivated by the success of the recently proposed method of anomaly cancellation to derive Hawking fluxes from black hole horizons of spacetimes in various dimensions, we have further extended the covariant anomaly cancellation method shortly simplified by Banerjee and Kulkarni to explore the Hawking radiation of the $(3+1)$-dimensional charged rotating black strings and their higher dimensional extensions in anti-de Sitter spacetimes, whose horizons are not spherical but can be toroidal, cylindrical or planar, according to their global identifications. It should be emphasized that our analysis presented here is very general in the sense that the determinant of the reduced $(1+1)$-dimensional effective metric from these black strings need not be equal to one ($\sqrt{-g} \neq 1$). Our results indicate that the gauge and energy–momentum fluxes needed to cancel the $(1+1)$-dimensional covariant gauge and gravitational anomalies are compatible with the Hawking fluxes. Besides, thermodynamics of these black strings are studied in the case of a variable cosmological constant.

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Keywords: Covariant anomaly; Hawking radiation; Black string; Thermodynamics

1. Introduction

Hawking's discovery [1] that a black hole is not completely black but can emit radiation from its horizon is an interesting and significant quantum effect arising in the background spacetime with an event horizon. This effect is named as the famous Hawking radiation. It plays an important role in catching on some clues to the complete theory of Quantum Gravity. This fact provides a strong motivation for understanding the essence of Hawking radiation since Hawking discovered it more than thirty years ago. During the past years, one prominent success is that a lot of new methods to derive Hawking radiation have been developed. For example, Robinson and Wilczek (RW) [2] recently presented a novel method to derive the Hawking temperature of a Schwarzschild black hole via the cancellation of $(1+1)$-dimensional consistent gravitational anomaly. Subsequently, this work was extended to the cases of a charged black hole [3] and rotating charged black holes [4] by considering both gauge and gravitational anomalies. Following this method, a lot of work [5–15] appeared soon. In Refs. [10], the original work of [2,3] was generalized to the general case where the metric determinant $\sqrt{-g} \neq 1$, and a peculiar case in which $\sqrt{-g}$ vanishes at the horizon has also been investigated [11] in details. As far as the case of non-spherical topology is concerned, only a little work [8,10] appeared so far. Therefore it is of special interest to further investigate Hawking radiation of black objects with non-spherical topology in other cases.

However, the original anomaly cancellation method proposed in Refs. [2,3] encompasses not only the consistent anomaly but also the covariant one. Quite recently, Banerjee and Kulkarni [12] suggested to simplify this model by considering only the covariant gauge and gravitational anomalies. Their simplification further cleans the description of the anomaly cancellation method totally in terms of the covariant expressions,

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thus making the analysis more economical and conceptually cleaner. An extension of their work to the case where the determinant $\sqrt{-g} \neq 1$ was done in [13]. Based upon these developments, some direct applications soon appeared in [14,15].

On the other hand, with the discovery of anti-de Sitter/conformal-field-theory (AdS/CFT) correspondence [16], it is of great interest to consider rotating charged generalizations of black holes in AdS spaces. The correspondence between the supergravity in asymptotically AdS spacetimes and CFT makes it possible to get some insights into the thermodynamic behavior of some strong coupling CFTs by studying thermodynamics of the asymptotically AdS spacetimes. Specifically speaking, according to the AdS/CFT correspondence, rotating black holes in AdS spaces are dual to certain CFTs in a rotating space, while the charged ones are dual to CFTs with a chemical potential.

What is more, higher dimensional generalizations (with or without a cosmological constant) of rotating black holes and their properties have attracted considerable attention in recent years, in particular, in the context of string theory, and with the advent of brane-world theories [17], raising the possibility of direct observation of Hawking radiation and of as probes of large spatial extra dimensions in future high energy colliders. The recent brane-world scenarios that predict the emergence of a TeV-scale gravity in the higher dimensional theory have opened the door towards testing Hawking effect and exploring extra dimensions through the decay of mini black holes to high energy particles.

Owing to the above two aspects, therefore it is of great importance to study Hawking radiation of higher dimensional AdS spacetimes, especially those of non-spherical topology. In this article, we will employ the covariant anomaly method to investigate Hawking radiation of the $(3+1)$-dimensional charged rotating black strings in AdS spacetimes and their higher dimensional extensions [18]. Our aim is to derive Hawking fluxes from these black strings via covariant gauge and gravitational anomalies. Our motivation is also due to the fact that the horizon topology of these black strings is not spherical but can be toroidal, cylindrical or planar, depending on their global identifications. It should be pointed that our covariant anomaly analysis is very general because the determinant of the reduced $(1+1)$-dimensional effective metric need not be equal to one ($\sqrt{-g} \neq 1$). Our results show that the gauge and energy–momentum fluxes needed to cancel the $(1+1)$-dimensional covariant gauge and gravitational anomalies are in agreement with the Hawking thermal fluxes. In addition, thermodynamics of these black strings are studied in the case of a variable cosmological constant.

Our Letter is organized as follows. In Section 2, we shall investigate thermodynamics of the $(3+1)$-dimensional black string and generalize the first law to the case of a variable cosmological constant. Then we derive the fluxes of the charge and energy–momentum tensor via covariant anomalies. A parallel analysis in higher dimensions is presented in Section 3, where we have investigated Hawking radiation from the higher-dimensional black strings. Section 4 is our summary. Appendix A is supplemented to address the non-uniqueness of two-dimensional effective and physically equivalent metrics in the process of dimensional reduction, taken the four-dimensional case as an explicit example.

2. Hawking radiation of the $(3+1)$-dimensional black string via covariant anomalies

We start with the action of Einstein–Maxwell theory in d-dimensional spacetimes with a negative cosmological constant, which reads

$$S_d = -\frac{1}{16\pi}\int d^d x\,\sqrt{-g}\left[R + \frac{(d-1)(d-2)}{l^2} - F^{\mu\nu}F_{\mu\nu}\right], \tag{1}$$

where l is the radius of AdS spaces. Under this action, a $(3+1)$-dimensional rotating charged black string solution in asymptotically AdS spaces was presented in [18]. In terms of Boyer–Lindquist-like coordinates, its line element and the corresponding gauge potential take the following forms

$$ds^2 = -f(r)(\Xi\,dt - a\,d\phi)^2 + \frac{dr^2}{f(r)} + \frac{r^2}{l^4}\left(a\,dt - \Xi l^2\,d\phi\right)^2 + r^2\,dz^2, \tag{2}$$

$$A = \frac{Q}{r}(\Xi\,dt - a\,d\phi), \tag{3}$$

where

$$f(r) = \frac{r^2}{l^2} - \frac{2M}{r} + \frac{Q^2}{r^2}, \qquad \Xi = \sqrt{1 + \frac{a^2}{l^2}}, \tag{4}$$

in which M, Q, and a are parameters related to the mass, the charge, and the angular momentum, respectively. When the coordinate z assumes the values $-\infty < z < +\infty$, the metric (2) describes a stationary black string with a cylindrical horizon. On the other hand, if the coordinate z is compactified to the region $0 \leqslant z < 2\pi$, one can get a closed black string with a toroidal horizon.

We first investigate thermodynamics of the black string. There exists an event horizon at r_+, which is the largest positive root of the equation $f(r) = 0$. The angular velocity and electrostatic potential of the horizon are

$$\Omega = \frac{a}{\Xi l^2}, \qquad \Phi = \frac{Q}{\Xi r_+}. \tag{5}$$

The Hawking temperature $T = \kappa/(2\pi)$ can be obtained via the surface gravity on the horizon, which is given by

$$\kappa = \frac{f'(r_+)}{2\Xi} = \frac{3r_+^4 - Q^2 l^2}{2\Xi l^2 r_+^3}. \tag{6}$$

For convenience, we assume the length of the black string along the z-direction is one. Then the Bekenstein–Hawking entropy for the horizon is $S = \pi\Xi r_H^2/2$. It can be verified that the mass, the angular momentum and the electric charge

$$\mathcal{M} = \frac{1}{4}\left(3\Xi^2 - 1\right)M, \qquad \mathcal{J} = \frac{3}{4}\Xi Ma, \qquad \mathcal{Q} = \frac{1}{2}\Xi Q, \tag{7}$$