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Interpenetrating subspaces as a funnel to extra space

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ABSTRACT

New solution for two interpenetrating universes is found. Higher derivative gravity acting in 6-dimensional space is the basis of the study that allows to obtain stable solution without introducing matter of any sort. Stability of the solution is maintained by a difference between asymptotic behavior at spacial infinities. For an external observer such a funnel looks similar to a spherical wormhole.

1. Introduction

The theory of gravity provides us with great variety of nontrivial objects. Most known of them are black holes and wormholes. The existence of black holes acquires substantial observational background nowadays. Wormholes are considered as a hypothetical way to pass from one large space to another: a property which distinguishes traversable wormholes from black holes [1,2]. Other stable topological configurations of space (geons) are considered as particle-like objects possessing a mass and a charge. The problem of a topology changes is discussed in [3]. A Lorentzian wormhole is a space-time whose spatial sections contain two asymptotically flat regions joined by a "throat". Most of the literature is devoted to 4-dim wormholes, however there is substantial branch concerning spaces of an arbitrary dimensionality, see e.g. [4]. Lorentzian wormholes embedded in the de Sitter space are discussed in [5–7].

In this paper, new solution with properties similar to those of a wormhole is discussed. The result is based on the extra space idea with a dimensionality $D > 5$ and the gravity with higher derivatives. Multidimensional wormholes are also discussed in the literature see e.g. [8]. The interest in $f(R)$ theories is motivated by inflationary scenarios starting from the pioneering work of Starobinsky [9]. A number of viable $f(R)$ models in 4-dim space that satisfy the observable constraints are proposed in Refs. [10–12]. Also, substantially new results may be obtained on the basis of $f(R)$-theories of gravity, see [17,20] and references therein.

In this paper it is supposed that our Universe is described by a D-dim space ($D > 4$) with a topology $T \times \mathbb{V}_{D-1}$. Its volume was comparable with unity in the Planck units at the moment of its origination from the space time foam. In the following three of these dimensions grew while others remained compact and/or small. It seems reasonable to suppose that the choice was made accidentally depending on initial conditions, see e.g. [21]. It is commonly accepted that our Universe belongs to only one of such 3-dim subspaces. If it is not true new space geometry caused by nontrivial boundary conditions at infinity is formed. Its structure is studied below. For an external observer the solution looks like a wormhole though its internal geometry is different.

2. Boundary conditions and funnel geometry

Let us start with metric

$$ds^2 = G_{\mu\nu} dZ^\mu dZ^\nu + G_{ab} dZ^a dZ^b \tag{1}$$

A lot of literature (see [22] for review) is devoted to study this metric. One of the simplest geometries described by metric (1) is the direct products $M_4 \times V_{D-4}$ of the Minkowski space and a $(D-4)$-dim extra space with metric

$$\begin{aligned} &G_{\mu\nu} = diag(1, -1, -Z_2^2, -Z_2^2 \sin^2 Z_3), \\ &G_{ab} = r_0 \cdot diag(-1, -\sin^2 Z_6, -\sin^2 Z_6 \sin^2 Z_7, \ldots), \\ &\mu, \nu = 1, 2, 3, 4 \quad a, b = 5, 6, \ldots, D, \end{aligned} \tag{2}$$

where r_0 is a radius of $(D-4)$-dim sphere with coordinates $Z_5, \ldots, Z_D$. The coordinates Z_1, Z_2, Z_3, Z_4 $(-\infty < Z_2 < \infty,\ 0 < Z_3 < \pi,\ 0 < Z_4 < 2\pi)$ describe the extended Minkowski space with the Ricci scalar $R_4 = 0$.

Let us consider more interesting case with the 4-dim metric depending on a single coordinate Z_2. It holds if boundary conditions at $Z_2 \to +\infty$ and at $Z_2 \to -\infty$ differ from each other. More definitely, suppose that first condition at $Z_2 \to +\infty$ coincides with static geometry (2). The subspace described by space coordinates 2, 3, 4 is assumed to be large.

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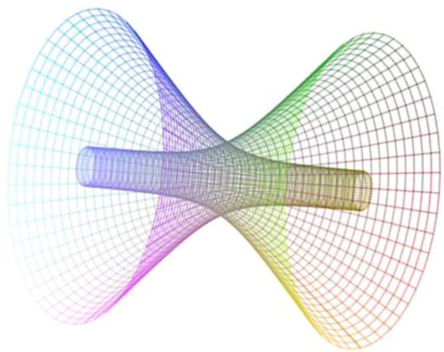

Fig. 1. Interpenetrating spaces in the spherical coordinates look like two intersecting funnels.

Another boundary condition at $Z_2 \to -\infty$

$$G_{\mu\nu}(Z_2 \to -\infty) = diag(1, -1, -Z_2^2, -Z_2^2 \sin^2 Z_6),$$
$$G_{ab}(Z_2 \to -\infty) = r_0 \cdot diag(-1, -\sin^2 Z_3, -\sin^2 Z_3 \sin^2 Z_4, ...),$$
$$\mu, \nu = 1, 2, 6, 7 \quad a, b = 5, 3, 4, 8..., D, \tag{3}$$

relates to another static large subspace with space coordinates 2, 6, 7.

Both boundary conditions (2) and (3) represent similar geometries with index permutation. Nevertheless the physical volume $v(Z_2)$ of the 2-dim subspace described be coordinates Z_3, Z_4 is different at $Z_2 \to \pm\infty$. Therefore a nontrivial solution connecting asymptotic regions $Z_2 \to +\infty$ and $Z_2 \to -\infty$ should exist in analogy with well known kink solutions [23].

One of the possible forms of such a space is shown in Fig. 1. This form is confirmed by a numerical simulation discussed below. If an observer moves along the Z_2-coordinate and intersects a point $Z_2 = 0$ she/he finds out an increasing of one large subspace and decreasing of another. Far from the transition at $Z_2 = 0$ both of these subspaces are described by the Minkowski geometry.

3. Funnel-like solution

3.1. Direct product of the Minkowski space and compact extra space

Let us specify a geometry of the space and consider the space $\mathbb{V}_D$ with $D = 6$ and metric in the form

$$ds^2 = e^{2\alpha(u)}dt^2 - du^2 - e^{2\beta_1(u)}G_{1,ab}dy^a dy^b - e^{2\beta_2(u)}G_{2,mn}dz^m dz^n \tag{4}$$

where $-\infty < u < \infty$. There are three independent functions – $\beta_1(u)$, $\beta_2(u)$ and the redshift function $\alpha(u)$. The variable u is a proper distance coordinate. The 2-dim subspaces $\mathbb{W}_{1,2}$ are described by coordinates y_a, z_m ($a = 3, 4$; $m = 5, 6$) and represent two spheres of radius $r_1(u) = e^{\beta_1(u)}$ and $r_2(u) = e^{\beta_2(u)}$.

The action is supposed contains the higher order derivatives of metric in the form

$$S = \frac{m_D^4}{2}\int d^6 Z\sqrt{|G|}\left[F(R) + c_1 R_{AB}R^{AB}\right]. \tag{5}$$

$$F(R) = R + cR^2 - 2\Lambda \tag{6}$$

Here c, c_1 and Λ are physical parameters of order m_D. By common views, higher order curvature terms appear due to quantum corrections, and it seems natural to include the Ricci tensor squared $R_{AB}R^{AB}$ and the term $\sim R^2$ on equal footing.

To make speculation as clear as possible suppose that the space $\mathbb{V}_D$ represents direct product of the 4-dim "large" space with coordinates t, u, y^1, y^2 and 2-dim sphere of radius $e^{\beta_2(u)}$, see (4). Due to the extremal smallness of the cosmological constant, we neglect its value, $\Lambda_0 = 0$. As is shown below this approximation imposes a condition to the Lagrangian parameters c, c_1 and Λ.

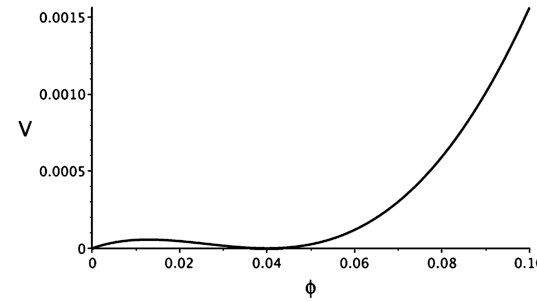

Fig. 2. The plot of the potential density V. The Lagrangian parameters $c = 5$, $\Lambda = 0.01$, $c_1 = -27$.

It is well known that the F(R) theory can be cast in the form of Einstein–Hilbert theory with a potential for the effective scalar-field degree of freedom [13–15] which strongly facilitate an analysis. Another method for the same purpose is developed in [16]. Both of these methods include the conformal transformation which holds only if $F'(R) \neq 0$. This condition has a profound basis. Indeed, as was shown in [18,19], theories of F(R) gravity are unstable at the hypersurface $F'(R) = 0$.

To proceed, let us use the method of slow motion [16]. More definitely, consider the limit

$$R_{(4)} \ll R_{(2)} \tag{7}$$

and assume that the metric tensor g_{AB} varies slowly with the coordinate u. After some calculations [16,17] we obtain the effective action in the Einstein frame

$$S_{eff} = \frac{v_2}{2}\int d^4x(signF')\left[R_{(4)} + \frac{k(\phi)}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)\right] \tag{8}$$

$$k(\phi) = \frac{1}{\phi}\left[3\phi^2\left(\frac{F''}{F}\right)^2 - 2\phi\left(\frac{F''}{F}\right) + 2\right] \tag{9}$$

$$V(\phi) = -sign(1 + 2c\phi)\frac{1}{2}\frac{|\phi|[(c + c_1/2)\phi^2 + \phi - 2\Lambda]}{(1 + 2c\phi)^2}. \tag{10}$$

Here and in the following $m_D = 1$ and the Planck mass $M_{Pl} = \sqrt{v_2}$. v_2 is the volume of 2-dim sphere of unit radius.

The potential density $V(\phi)$ represented in Fig. 2 depends on the scalar field which is connected to the Ricci scalar $R_{(2)}$ of the extra space, $\phi(u) \equiv R_{(2)} = 2e^{-2\beta(u)}$. The presence of the potential minimum indicates stationarity of extra space of constant curvature.

Necessary conditions for the cosmological constant be equal zero have the form

$$V(\phi_M) = 0, \quad V'(\phi_M) = 0. \tag{11}$$

These equations fix the field

$$\phi_M = 4\Lambda \tag{12}$$

and give the connection between the physical parameters

$$\Lambda = \frac{-1}{4(2c + c_1)}. \tag{13}$$

The radius r_0 of the extra space is expressed in the form

$$r_0 = e^{\beta_M} = \sqrt{\frac{2}{\phi_M}} = \frac{1}{\sqrt{2\Lambda}} \tag{14}$$