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Inverse magnetic catalysis from the properties of the QCD coupling in a magnetic field

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ABSTRACT

We compute the vacuum one-loop quark–gluon vertex correction at zero temperature in the presence of a magnetic field. From the vertex function we extract the effective quark–gluon coupling and show that it grows with increasing magnetic field strength. The effect is due to a subtle competition between the color charge associated to gluons and the color charge associated to quarks, the former being larger than the latter. In contrast, at high temperature the effective thermo-magnetic coupling results exclusively from the contribution of the color charge associated to quarks. This produces a decrease of the coupling with increasing field strength. We interpret the results in terms of a geometrical effect whereby the magnetic field induces, on average, a closer distance between the (electrically charged) quarks and antiquarks. At high temperature, since the effective coupling is proportional only to the color charge associated to quarks, such proximity with increasing field strength makes the effective coupling decrease due to asymptotic freedom. In turn, this leads to a decreasing quark condensate. In contrast, at zero temperature both the effective strong coupling and the quark condensate increase with increasing magnetic field. This is due to the color charge associated to gluons dominating over that associated to quarks, with both having the opposite sign. Thus, the gluons induce a kind of screening of the quark color charge, in spite of the quark–antiquark proximity. We discuss the implications for the inverse magnetic catalysis phenomenon.

The properties of strongly interacting matter in the presence of magnetic fields, as found in recent lattice QCD (LQCD) determinations, exhibit intriguing characteristics. In a thermal environment, at and above the transition temperature for deconfinement/chiral symmetry restoration, the magnetic field hinders the formation of the quark condensate [1] and makes the critical temperature decrease with increasing field strength [2]. This behavior is dubbed *inverse magnetic catalysis*. In contrast, the vacuum ($T=0$) condensate grows with the magnetic field strength. As the temperature increases near, but below the transition temperature, the condensate begins to grow for weak fields reaching a maximum value, smaller than for $T=0$ and the same field strength. Subsequently, the condensate decreases with increasing field strength. This growth of the quark condensate with magnetic field strength corresponds to magnetic catalysis. Overall, this behavior indicates that the strength of the QCD interaction at $T=0$ is enhanced by the magnetic field, thus strengthening the binding of quark–antiquark pairs that make up the condensate. However, as the temperature increases, such binding becomes weaker. When the temperature reaches the transition region the magnetic field dominates the interaction, quenching monotonically the binding for all field strengths. The search for an explanation of such properties has attracted the attention of a great deal of research over the last years [3,4]. A possible way to look at this effect has been casted in terms of the competition between the valence and the sea contributions to the quark condensate. It has been argued that at $T=0$ both contributions are growing as a function of eB. However, around the critical temperature T_c the valence contribution is still increasing whereas the sea contribution decreases, as a function of

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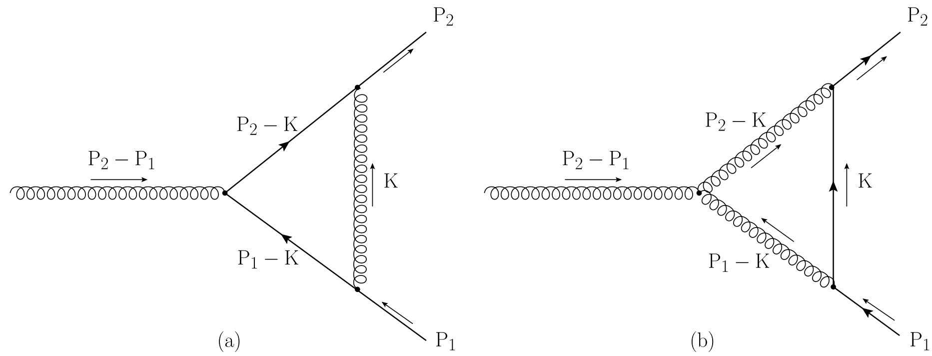

Fig. 1. Feynman diagrams contributing to the magnetic dependence of the quark–gluon vertex. Diagram (a) corresponds to a QED-like contribution whereas diagram (b) corresponds to a pure QCD contribution.

eB. This seemingly results in a decrease of T_c as a function of eB. For recent reviews see [5,6].

On general grounds a magnetic field interacting with electrically charged particles acts as an *ordering agent*. In other words, the motion of virtual or real charges takes place around the magnetic field lines. This ordered motion has an important geometrical consequence: charged particles are closer to each other on average. When the intensity of the magnetic field increases, so does the proximity between charges. As is well known, due to asymptotic freedom, the closer strongly interacting particles are, the weaker the interaction. However strongly interacting matter, either at zero or at finite temperature, is not only made out of quarks and antiquarks but also of electrically neutral gluons. If the geometrical effect produced by the magnetic field were related to inverse magnetic catalysis, then at low temperatures the color interactions produced by gluons should dominate, while quarks would take over at high temperatures.

An important clue on the properties of strongly interacting matter in the presence of a magnetic field has been provided in [7] for the case of high temperature. There it was shown that under such conditions the quark–gluon effective coupling decreases with the field intensity and that the color charge contribution from the gluons cancels exactly. Furthermore, the magnetic field-dependent vertex correction satisfies a Ward-like identity involving the magnetic field dependent quark self-energy. This means that at high temperature color dynamics is dominated by quarks. This behavior can be understood in terms of the geometrical picture whereby the proximity between electric charges induced by the magnetic field dominates the color interaction. An outstanding question is whether this picture holds also at $T = 0$, namely, whether under such circumstances the strength of the color interaction becomes, instead, gluon dominated.

In this paper we compute the magnetic field contribution to the quark–gluon vertex in vacuum and show that, indeed, the strong interaction becomes dominated by the contribution of the electrically neutral gluons. This generates an effective coupling that grows with increasing field strength, in contrast with the high-temperature result. Recall that inverse magnetic catalysis can also be quantified in terms of the properties of the quark condensate as a function of the magnetic field. Since the condensate is a measure of the strength of the bound between either vacuum ($T = 0$) or thermal ($T \neq 0$) quark–antiquark pairs and α_s is a measure of the strength of the interaction between these quark–antiquark pairs, both quantities represent the strength of the quark–antiquark binding. We show that a mechanism that can help understand inverse magnetic catalysis consists on pursuing the relation between the properties of α_s as a function of the magnetic field and the condensate. In this context we recall that several calculations that address the behavior of the quark condensate in the presence of a magnetic field, coincide in that the condensate is an increasing function of the field strength [8]. Both, the coupling constant and the condensate, should behave similarly as a function of the field strength. We find that in the two extreme cases, namely, at high and zero T, they do. Here we do not address the details of how this change happens, which certainly require non-perturbative information for their description. However, by establishing that this change in the properties of α_s happens at these two extremes, we put forward a novel scenario to study inverse magnetic catalysis in terms of the thermomagnetic properties of the strong coupling constant.

We begin by considering the case of a magnetic field pointing along the $\hat{z}$ direction. In a magnetic background, the fermion propagator in coordinate space can no longer be written as a simple Fourier transform of a momentum propagator but instead it is written as [9]

$$S(x,x') = \Phi(x,x') \int \frac{d^4p}{(2\pi)^4} e^{-ip\cdot(x-x')} S(p), \tag{1}$$

where $\Phi(x,x')$ is called the *Schwinger phase factor*. The translational invariant part of the propagator, $S(p)$, is given by

$$iS(p) = \int_0^\infty \frac{ds}{\cos(qBs)} e^{is\left(p_\parallel^2 - p_\perp^2 \frac{\tan(qBs)}{qBs} - m^2\right)} \times \left\{ \left[\cos(qBs) + \gamma_1\gamma_2 \sin(qBs)\right](m + \not{p}_\parallel) - \frac{\not{p}_\perp}{\cos(qBs)} \right\}, \tag{2}$$

where m and q are the quark mass and absolute value of the quark charge, in units of the electron charge, respectively. Hereafter we use the following definitions for the parallel and perpendicular components of the scalar product of any two vectors a^μ and b^μ

$$(a\cdot b)_\parallel = a_0b_0 - a_3b_3$$
$$(a\cdot b)_\perp = a_1b_1 + a_2b_2. \tag{3}$$

Fig. 1 shows the Feynman diagrams contributing to the quark–gluon vertex. Diagram (a) corresponds to a QED-like contribution whereas diagram (b) corresponds to the pure QCD contribution. The computation of these diagrams requires the fermion propagator given by Eq. (1), which involves the Schwinger phase factor $\Phi(x,x')$. It can be shown [7] that when only one or two fermion propagators are involved in this kind of triangle loop, the phase factor can be *gauged away* and we can just work with the translationally invariant part of the fermion propagators.

Since the effect we are after shows up already for small magnetic field strengths, we consider the case of a weak field for which the fermion propagator can be written as [10]