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Model discrimination in pseudoscalar-meson photoproduction

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ABSTRACT

To learn about a physical system of interest, experimental results must be able to discriminate among models. We introduce a geometrical measure to quantify the distance between models for pseudoscalar-meson photoproduction in amplitude space. Experimental observables, with finite precision, map to probability distributions in amplitude space, and the characteristic width scale of such distributions needs to be smaller than the distance between models if the observable data are going to be useful. We therefore also introduce a method for evaluating probability distributions in amplitude space that arise as a result of one or more measurements, and show how one can use this to determine what further measurements are going to be necessary to be able to discriminate among models.

1. Introduction

Nuclear and hadron physics have entered an era of high precision measurements from often very demanding experiments. In the planning stage, it is important to estimate the potential impact of a particular set of measurements. High impact experiments are ones in which there is a large potential for the data to constrain the models of the underlying physical processes of interest, typically by greatly reducing uncertainties in model parameters. An analysis of nucleon–nucleon scattering data, for example, with advanced statistical methods [1] allows one to infer the parameters and corresponding errors in nucleon–nucleon potentials. Statistical methods that are designed to reliably infer parameters from experimental data are, however, not necessarily optimized to estimate the potential impact of various combinations of possible experiments. In other words, *model discrimination* often requires different strategies than *parameter estimation* within models [2–4].

In this paper we lay out a framework that can be used to obtain estimates of the possible impact of (combinations) of polarization measurements in pseudoscalar-meson photoproduction from the nucleon (hereafter denoted as $\gamma N \to MB$). Information about the reaction amplitudes in a particular range of kinematics is the key to discriminating between two or more models. In imaging systems, the Rayleigh criterion is used to determine whether two or more light sources can be resolved from each other. We develop an analogue of this criterion which requires a measure of the distance between models in amplitude space, and a means of determining the characteristic spread of probability densities in amplitude space that result from measurement of observables. Amplitude space is defined as the parameter space of the reaction amplitudes. The latter are connected with the probability of two particles with given spin and four-momentum to interact with each other and end up in a well-defined final reaction channel.

Several models for the underlying reaction mechanisms of $\gamma N \to MB$ reactions are available. Some of the most common approaches are the coupled-channel (CC), isobar and hybrid isobar-Regge models. All of these aim to extract s-channel resonance content from experimental data. In most cases, model assumptions are required to describe other contributing mechanisms (referred to as "the background"). After decades of research, however, the precise underlying resonance content is still under debate. With the inclusion of more diverse and high-statistics experimental data, the list of known resonances of the Review of Particle Physics [5] has changed. A detailed knowledge of the reaction amplitudes as a function of kinematical variables should enable one to discriminate among various reaction models, but it is necessary to perform measurements of several $\gamma N \to MB$ polarization observables to access the reaction amplitudes.

At fixed kinematics, four complex reaction amplitudes determine the $\gamma N \to MB$ dynamics. The kinematics are fixed by the invariant mass W and the cosine of the center-of-mass (c.m.) scattering angle $\theta_{\text{c.m.}}$, and there is a one-to-one relation between $(W, \cos\theta_{\text{c.m.}})$ and the Mandelstam variables (s, t). It was suggested [6,7] that a selection of polarization measurements may lead to a situation where all reaction amplitudes are known to the extent

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that the outcome of any future experiment could be predicted. In Ref. [7] it was shown that eight well-chosen observables suffice to unambiguously determine the amplitudes. One refers to a such a combination of observables as a "complete set". However, this is only true in a mathematical sense, and it has been established that there is no such thing as complete sets when dealing with data with finite error bars [8–12].

Two categories can be distinguished for polarization observables: single-polarization ($\mathcal{S} = \{\Sigma$ (beam), T (target), P (recoil)$\}$) where only one of the initial and final state particles is polarized, and double-polarization that require two polarized particles. The latter category can be subdivided into three categories: beam–recoil ($\mathcal{BR} = \{C_x, C_z, O_x, O_z\}$), beam–target ($\mathcal{BT} = \{E, F, G, H\}$) and target–recoil ($\mathcal{TR} = \{T_x, T_z, L_x, L_z\}$) observables [13]. These are connected to the reaction amplitudes through bilinear relations (see e.g. Ref. [8]). We note that in practice, experiments are configured to have beam polarization, target polarization, the ability to determine recoil polarization or some combination thereof. Each of these experimental configurations are sensitive to different combinations of "observables", and so not all observables can be measured in isolation [14].

Models that are fitted to the published observables, can in fact have very different reaction amplitudes. An example is the $\mathcal{BT}$ double polarization observable E in $\vec{\gamma}\vec{p} \to \pi^+ n$ that was measured recently [15]. Despite the availability of data for other observables, the existing $\gamma p \to \pi^+ n$ models predicted a large range of values of E at similar kinematic points (see Fig. 3 in Ref. [15]), pointing to substantial differences among the models at the amplitude level. The overall or "global" performance of two models can be compared by averaging their least squared-distance to the measurements over all experimentally probed kinematics. More restrictive is a "local" model discrimination, where models are compared at specific kinematics (s, t). A partial-wave analysis parametrizes the $\cos\theta_{\mathrm{c.m.}}$ dependence of the reaction amplitudes at fixed s and can be regarded as an analysis technique that falls in between "local" and "global". In this work, we focus on the most local (and completely model-independent) form of amplitude analysis, but we note that in practice it is probable that model comparison will be done with partial wave analyses. The question that we aim to address is what kind of experimental results do we need to be able to discriminate between various models at specific kinematics.

In this work we use transversity amplitudes (TA), where particle spins are quantized in a transverse basis. The TA have so-called "optimally simple" relations [16] to the observables, in which the single-polarization observables depend on the amplitude moduli only [8,9]. The transition amplitude $\boldsymbol{T}^{\mathcal{B}}_{\mathcal{T},\mathcal{R}}$ for a fixed photon $\mathcal{B}$, nucleon $\mathcal{T}$ and baryon $\mathcal{R}$ polarization, reads

$$\boldsymbol{T}^{\mathcal{B}}_{\mathcal{T},\mathcal{R}} \equiv \overline{u}^{\mathcal{R}}_{B} \epsilon^{\mu}_{\mathcal{B}} \hat{J}_{\mu} u^{\mathcal{T}}_{N}. \tag{1}$$

The u_B (u_N) denotes the recoil (target) Dirac spinor, $\hat{J}^{\mu}$ the interaction current and $\epsilon^{\mu}_{\mathcal{B}}$ the γ-polarization four-vector. For a linearly polarized photon along the x or y axis one has $\epsilon^{\mu}_{\mathcal{B}=x} = (0, 1, 0, 0)$, $\epsilon^{\mu}_{\mathcal{B}=y} = (0, 0, 1, 0)$. The transversity basis is defined as

$$b_1 = \boldsymbol{T}^{y}_{+y,+y},\ b_2 = \boldsymbol{T}^{y}_{-y,-y},\ b_3 = \boldsymbol{T}^{x}_{-y,+y},\ b_4 = \boldsymbol{T}^{x}_{+y,-y}. \tag{2}$$

The $\mathcal{R} = \pm y$ ($\mathcal{T} = \pm y$) denotes a recoil (target) spin quantum number $\pm\frac{1}{2}$ along the y direction.

In order to quantify the differences between the predictions for the magnitude of the cross sections between the models A and B, we introduce the asymmetry

$$\mathcal{A}[A, B](W, \cos\theta_{\mathrm{c.m.}}) = \left| \frac{\frac{d\sigma}{d\Omega}(A) - \frac{d\sigma}{d\Omega}(B)}{\frac{d\sigma}{d\Omega}(A) + \frac{d\sigma}{d\Omega}(B)} \right|. \tag{3}$$

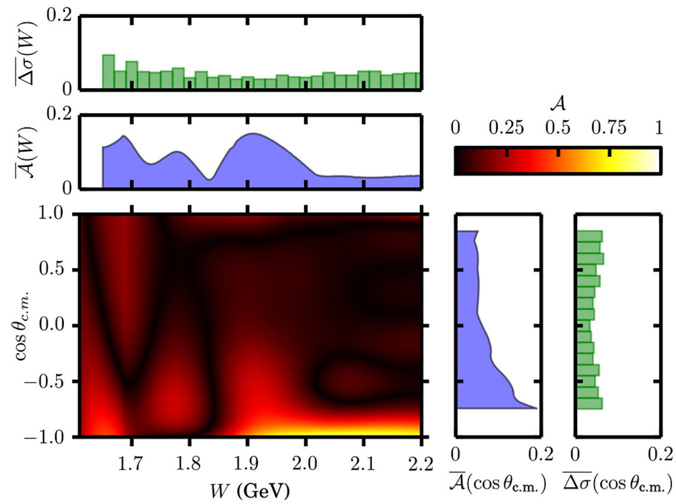

Fig. 1. The energy and angular dependence of the $\mathcal{A}$ defined in Eq. (3) between the BoGa and RPR-2011 models for $\gamma p \to K^+\Lambda$. Also shown are the average $\overline{\mathcal{A}}(\cos\theta_{\mathrm{c.m.}}) = \frac{1}{b-a}\int_a^b dW\, \mathcal{A}(W, \cos\theta_{\mathrm{c.m.}})$ [a similar formula holds for $\overline{\mathcal{A}}(W)$] in "realistic kinematics" (RK). Realistic kinematics refers to kinematics accessible with reasonable statistics by existing experimental facilities and is determined by the ranges $W \geq 1.65$ GeV and $-0.75 \leq \cos\theta_{\mathrm{c.m.}} \leq 0.85$. The $\overline{\Delta\sigma}(W)$ and $\overline{\Delta\sigma}(\cos\theta_{c.m.})$ are obtained by evaluating the $\gamma p \to K^+\Lambda$ measurements for $\frac{d\sigma}{d\Omega}$. We calculate the relative error $\left(\Delta\frac{d\sigma}{d\Omega}\right)/\frac{d\sigma}{d\Omega}$ on an equidistant $(W, \cos\theta_{\mathrm{c.m.}})$ grid, using the data from the CLAS Collaboration [25,26]. For each kinematic bin we collect the available $\frac{d\sigma}{d\Omega}$ data and run a bootstrap algorithm to estimate the error $\left(\Delta\frac{d\sigma}{d\Omega}\right)$. To compute $\overline{\Delta\sigma}(W)$, for example, we average over the covered $\cos\theta_{\mathrm{c.m.}}$ range at given W.

In what follows we use the representative Bonn–Gatchina (BG2014-02) [17] (BoGa) and hybrid Regge-plus-Resonance [18] (RPR-2011) models for $\gamma p \to K^+\Lambda$ to set the scale of the introduced measure. The BoGa model is a highly sophisticated coupled-channel model. The RPR-2011 model is a hybrid Regge-isobar model for $\gamma p \to K^+\Lambda$ with very low number of parameters. Both models are fitted to a large data set of cross sections, a sizable set of single-polarization observables (mostly P) and a limited number of double-polarization observables. The BoGa and RPR-2011 models parametrize the $\gamma p \to K^+\Lambda$ background very differently at low energies. Fig. 1 shows $\mathcal{A}[A = \text{BoGa}, B = \text{RPR-2011}](W, \cos\theta_{\mathrm{c.m.}})$. Both models produce comparable cross sections at forward $\theta_{\mathrm{c.m.}}$. The results for $\overline{\mathcal{A}}(\cos\theta_{c.m.})$ indicate that the deviations between BoGa and RPR-2011 grow with increasing $\theta_{\mathrm{c.m.}}$. This reflects the fact that the description of the background (which requires only a few parameters) in the RPR-2011 model is physically less justified at backward angles [19].

At extremely backward $\theta_{\mathrm{c.m.}}$ and in the threshold region, the measurements typically come with low statistics. Good experimental statistics are obtained for $W \geq 1.65$ GeV and $-0.75 \leq \cos\theta_{\mathrm{c.m.}} \leq 0.85$. In this selected "realistic kinematics" (RK) the $\mathcal{A}$[BoGa, RPR-2011] typically clusters around 0.1–0.2. The experimental equivalent of the asymmetry $\mathcal{A}$ is the relative error $\left(\Delta\frac{d\sigma}{d\Omega}\right)/\frac{d\sigma}{d\Omega}$. The results are included in Fig. 1 and are systematically of the order 0.06 in both W and $\cos\theta_{\mathrm{c.m.}}$. Comparing the $\overline{\mathcal{A}}(\cos\theta_{\mathrm{c.m.}})$ for (BoGa, RPR-2011) with the experimental figure-of-merit $\overline{\Delta\sigma}(\cos\theta_{c.m.})$, leads us to conclude that the available experimental information from cross-section measurements in the $\gamma p \to K^+\Lambda$ channel is already contained in the BoGa and RPR-2011 models. As a result, further measurements of $\frac{d\sigma}{d\Omega}$ for $\gamma p \to K^+\Lambda$ are unlikely to provide information to further discriminate between the assumptions underlying the "BoGa" and "RPR-2011" models.