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Unparticle contribution to the hydrogen atom ground state energy

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ABSTRACT

In the present work we study the effect of unparticle modified static potentials on the energy levels of the hydrogen atom. By using Rayleigh–Schrödinger perturbation theory, we obtain the energy shift of the ground state and compare it with experimental data. Bounds on the unparticle energy scale $\Lambda_{\mathcal{U}}$ as a function of the scaling dimension $d_{\mathcal{U}}$ and the coupling constant λ are derived. We show that there exists a parameter region where bounds on $\Lambda_{\mathcal{U}}$ are stringent, signaling that unparticles could be tested in atomic physics experiments.

1. Introduction

Unparticle physics is an extension of the Standard Model, consisting in the possibility of having a non-trivial scale invariant, yet undiscovered sector of particle physics.

At first sight, unparticles appear as a generalization of neutrinos because they share the following properties: scale invariance and only very weak interaction with other fields. Neutrinos enjoy the first property to a good approximation, although their oscillations disclose a small non-zero mass. The second property is a general requirement of any hypothetical particle sector because we want it to be, to a certain extent, hidden from current observations. On closer inspection, however, we find that unparticles differ drastically from neutrinos. Since we do not restrict the unparticle fields to be massless, we can no longer speak in terms of particle numbers as in the conventional manner. The unparticle field is controlled by a canonical scaling dimension $d_{\mathcal{U}}$ which is in general a non-integer number. Due to the unusual character, one refers to the matter described by such a theory as *unlike particles*, or unparticle stuff.

After Georgi's seminal paper [1] unparticle effects have been explored in many areas spanning collider physics [2–8], muonic atoms [9], gauge and Higgs interactions [10,11], cosmology and astrophysics [12,13], AdS/CFT correspondence [14] as well as gravity short scale deviations [15,16] and black holes [17–21]. Unparticles also play a crucial role in the fractal properties of a quantum spacetime. A new fractality indicator, called un-spectral dimension, has recently been proposed to address the case of a random-walker problem in terms of an unparticle probe [22]. When the manifold topological dimension is 2, the un-spectral dimension turns out to be $2d_{\mathcal{U}}$, *i.e.*, it depends only on the scaling dimension $d_{\mathcal{U}}$. This fact explains the complete "fractalizazion" of the event horizon of un-gravity black holes [19,20], as well as of metallic plates for the Casimir effect in the presence of an un-photon field [23]. Finally, unparticles have been proposed to explain some anomalies in currents flowing in super-conductors [24] and transport phenomena in cuprates [25].

In this paper we want to address one of the basic parameters of unparticle physics, *i.e.*, the value of $\Lambda_{\mathcal{U}}$, the typical energy scale of the theory. To achieve this goal, we consider the modifications of static potentials that emerge from virtual unparticle exchange. Specifically by considering the corrections to the Coulomb potential we calculate the deviations of the ground state energy of the hydrogen atom. We show that competitive bounds on $\Lambda_{\mathcal{U}}$ can be derived by a comparison with experimental data.

The paper is organized as follows. After a short review of the basic formalism of unparticle physics (Section 2), we present the calculation of the energy shift by a perturbative solution of the Schrödinger equation in the presence of an unparticle modified electrostatic potential (Section 3). Finally, in Section 4 we draw our conclusions.

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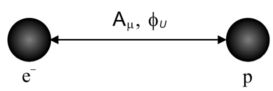

Fig. 1. Electron–proton interaction due to a vector field A_μ and an unparticle scalar field $\phi_\mathcal{U}$.

2. Unparticle physics and static potentials

We recap the basic motivations of unparticle physics along the lines of Georgi [1]. We start by considering that at some very high energy scale, the Standard Model is accompanied by an additional sector of Banks–Zaks fields ($\mathcal{BZ}$). The interaction between the two sectors takes place by exchange of mediating particles having a large mass scale $M_\mathcal{U}$. If our energy scale of interest falls below $M_\mathcal{U}$, we can apply effective field theory to integrate out the mediating field [26] and get the final interaction Lagrangian

$$\frac{1}{M_\mathcal{U}^k}\,\mathcal{O}_{\mathrm{SM}}\,\mathcal{O}_{\mathcal{BZ}} \tag{1}$$

where $\mathcal{O}_{\mathrm{SM}}$ denotes a Standard Model field operator of scaling dimension d_{SM} and $\mathcal{O}_{\mathcal{BZ}}$ is a Banks–Zaks field operator of scaling dimension $d_{\mathcal{BZ}}$. The factor $M_\mathcal{U}^{-k}$ guarantees the dimensional consistency of (1), being $k = d_{\mathrm{SM}} + d_{\mathcal{BZ}} - D$ and D is the spacetime dimension.

If the energy is further decreased to $\Lambda_\mathcal{U} < M_\mathcal{U}$, the Banks–Zaks fields undergo a dimensional transmutation and exhibit a scale invariant behavior with a continuous mass distribution. For energies lower than $\Lambda_\mathcal{U}$, the $\mathcal{BZ}$ sector becomes unparticle operators $\mathcal{O}_\mathcal{U}$. The matching conditions onto the Banks–Zaks operators are imposed at the energy scale $\Lambda_\mathcal{U}$ and determine the structure of the coupling between the Standard Model and the unparticle fields based on (1):

$$\frac{C_\mathcal{U}\,\Lambda_\mathcal{U}^{d_{\mathcal{BZ}}-d_\mathcal{U}}}{M_\mathcal{U}^k}\,\mathcal{O}_{\mathrm{SM}}\,\mathcal{O}_\mathcal{U} = \frac{\lambda}{\Lambda_\mathcal{U}^{d_{\mathrm{SM}}+d_\mathcal{U}-D}}\,\mathcal{O}_{\mathrm{SM}}\,\mathcal{O}_\mathcal{U} \tag{2}$$

where $d_\mathcal{U}$ is the scaling dimension of the unparticle operator $\mathcal{O}_\mathcal{U}$, $C_\mathcal{U}$ denotes a dimensionless constant and λ is a dimensionless coupling parameter defined by

$$\lambda = C_\mathcal{U}\left(\frac{\Lambda_\mathcal{U}}{M_\mathcal{U}}\right)^k < 1\,. \tag{3}$$

The inequality holds if $C_\mathcal{U} < 1$ and $d_{\mathcal{BZ}} > D - d_{\mathrm{SM}}$. Any experimental bound on the interaction allows for constraints on the unparticle parameter space, *i.e.*, $\Lambda_\mathcal{U}$, λ and $d_\mathcal{U}$. The scale hierarchy is given by 1 TeV $\leq \Lambda_\mathcal{U} < M_\mathcal{U} \leq M_{\mathrm{Pl}}$, where M_{Pl} is the Planck mass [21]. In the rest of the present paper we assume $D = 4$ as well as the customary interval $1 < d_\mathcal{U} < 2$ (cf. [2]). The case $d_\mathcal{U} = 1$ does not give rise to fractalizazion or other continuous dimension effects of unparticle physics.

Unparticles have been largely employed in context of static potential emerging from virtual particle exchange [11,15,16,20,27]. Such results are instrumental to the working hypothesis of the current investigation. In view of the analysis of the hydrogen atom, we consider an additional contribution to the Coulomb potential for the presence of unparticle exchange in the interaction between electron and proton (cf. Fig. 1). In general, we assume electrons and protons to carry unparticle charges λ_e and λ_p, respectively, and a scalar unparticle field $\phi_\mathcal{U}$. At the same time, both particles also possess electric charges $\pm e$ allowing them to couple to the photon field A_μ. Both interactions are independent of each other since we assume the effective couplings between unparticle stuff and photons to be negligible. Therefore the interaction Lagrangian can be written as

$$\mathcal{L}_{\mathrm{int}} = J^\mu A_\mu + \frac{1}{(\Lambda_\mathcal{U})^{d_\mathcal{U}-1}}\,J_\mathcal{U}\phi_\mathcal{U} \tag{4}$$

where $J^\mu = j(\vec{x})\,\delta_0^\mu$ and $J_\mathcal{U} = j_\mathcal{U}(\vec{x})$ and

$$j(\vec{x}) = -e\,\delta(\vec{x}-\vec{x}_\mathrm{e}) + e\,\delta(\vec{x}-\vec{x}_\mathrm{p}) \tag{5}$$

$$j_\mathcal{U}(\vec{x}) = \lambda_\mathrm{e}\,\delta(\vec{x}-\vec{x}_\mathrm{e}) + \lambda_\mathrm{p}\,\delta(\vec{x}-\vec{x}_\mathrm{p}). \tag{6}$$

Alternatively one can consider the interaction Lagrangian[1] for a vector unparticle field $(A_\mathcal{U})_\mu$ and derive the static potential much in the same way as in the scalar case. The two results do not differ apart from a global sign (see *e.g.* [16]). We recall, however, that conformal invariance can be lost for vector fields if $d_\mathcal{U} < 3$, although pure scale invariance can be preserved. For scalar unparticles the issue does not arise [26].

The expression for the unparticle interaction energy $V_\mathcal{U}$ between an electron and a proton in the static case reads [11,15,16,20,27]

$$V_\mathcal{U} = -\xi_{d_\mathcal{U}}\left(\frac{\lambda_\mathrm{e}\,\lambda_\mathrm{p}}{\Lambda_\mathcal{U}^{2d_\mathcal{U}-2}\,r^{2d_\mathcal{U}-1}}\right) \tag{7}$$

where the coefficient is $\xi_{d_\mathcal{U}} \equiv \frac{\sqrt{\pi}}{(2\pi)^{2d_\mathcal{U}}}\,\frac{\Gamma\left(d_\mathcal{U}-\frac{1}{2}\right)}{\Gamma(d_\mathcal{U})}$ and $r \equiv |\vec{x}_\mathrm{e}-\vec{x}_\mathrm{p}|$ denotes the distance between the charges.

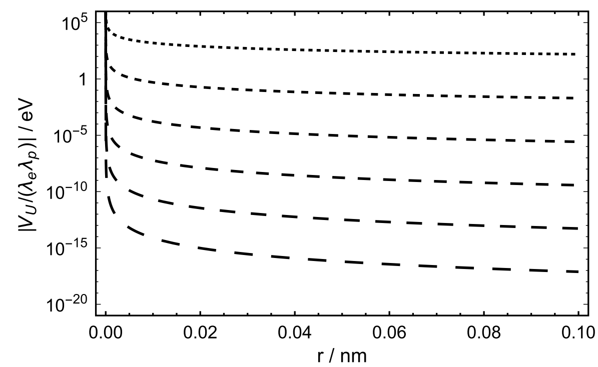

Fig. 2. Interaction energy per unit unparticle charge with respect to the radial distance r in the case $\Lambda_\mathcal{U} = 1$ TeV. The unparticle scaling dimension $d_\mathcal{U}$ is raised from $d_\mathcal{U} = 1.0$ to $d_\mathcal{U} = 2.0$ in steps of 0.2 corresponding to growing dash lengths.

For the typical size of the hydrogen atom we can estimate the energy correction with the help of Fig. 2. For $r = 0.05$ nm the energy shift per unit unparticle charge varies between 3×10^2 eV and 6×10^{-19} eV in the range $1 < d_\mathcal{U} < 2$ with $\Lambda_\mathcal{U} = 1$ TeV.

3. Unparticle effects in the hydrogen atom

The hydrogen atom is a well investigated system. In the nonrelativistic limit we can apply the Schrödinger formalism to describe electron dynamics in a static Coulomb potential of the proton, by using the particle reduced mass μ. The energy spectrum in Gaussian/natural units ($4\pi\epsilon_0 = \hbar = c = 1$) reads

$$E^{\mathrm{S}}_{\mathrm{th},\,n} = -\frac{\mu\,\alpha^2}{2n^2} \tag{8}$$

[1] The Lagrangian in such a case reads

$$\mathcal{L}_{\mathrm{int}} = J^\mu A_\mu + \frac{1}{(\Lambda_\mathcal{U})^{d_\mathcal{U}-1}}\,J^\mu_\mathcal{U}(A_\mathcal{U})_\mu$$

where $J^\mu_\mathcal{U} = j_\mathcal{U}(\vec{x})\,\delta^\mu_0$.