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Character of matter in holography: Spin–orbit interaction

Yunseok Seo [a], Keun-Young Kim [b], Kyung Kiu Kim [b,c], Sang-Jin Sin [a,*]

[a] Department of Physics, Hanyang University, Seoul 133-791, Republic of Korea
[b] School of Physics and Chemistry, Gwangju Institute of Science and Technology, Gwangju 500-712, Republic of Korea
[c] Department of Physics, College of Science, Yonsei University, Seoul 120-749, Republic of Korea

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ABSTRACT

Gauge/Gravity duality as a theory of matter needs a systematic way to characterise a system. We suggest a 'dimensional lifting' of the least irrelevant interaction to the bulk theory. As an example, we consider the spin–orbit interaction, which causes magneto-electric interaction term. We show that its lifting is an axionic coupling. We present an exact and analytic solution describing diamagnetic response. Experimental data on annealed graphite shows a remarkable similarity to our theoretical result. We also find an analytic formulas of DC transport coefficients, according to which, the anomalous Hall coefficient interpolates between the coherent metallic regime with ρ_{xx}^2 and incoherent metallic regime with ρ_{xx} as we increase the disorder parameter β. The strength of the spin–orbit interaction also interpolates between the two scaling regimes.

Overview and summary: Recently, the gauge/gravity duality [1–3] attracted much interests as a possible candidate for a reliable method to calculate strongly correlated systems. It is a local field theory in one higher dimensional space called "bulk", with a few classical fields coupled with anti-de Sitter (AdS) gravity. Since the strong coupling in the boundary is dual to a weak coupling in the bulk, the bulk fields can be considered as local order parameters of a mean field theory in the bulk. It also provided a new mechanism for instabilities in gravity language [4] which is relevant to the superconductivity [5,6] and the metal insulator transition [7]. However, as a theory for materials, it is still in lack of one essential ingredient, a way to distinguish one matter from the others. Although electron–electron interaction is traded for the gravity in the bulk, we still need to specify lattice–electron interactions to characterise the system. Without it, we would not know what system we are working for.

Naively one may try to introduce realistic lattice at the boundary to mimic the reality. However, its effects are mostly irrelevant in the infrared (IR) limit. In strong coupling limit where no quasiparticle exists, no Fermi surface (FS) exists either. Actually in the absence of the FS, it is almost impossible to write down any relevant *interaction* term in a local field theory in higher than 1+1 dimension.[1] Therefore non-local effect may be essential for any interesting physics in strongly interacting system. One interesting aspect of a holographic theory is that any local interaction in the bulk has non-local effect in the boundary [9]. Usually one characterises a many body system in continuum limit by a few interaction terms rather than the detail of structure. Therefore, to characterise a system in holographic theory, what we want to suggest is the dimensional-lifting, by which we mean promoting the "system characterising interaction" of the boundary theory to a term in the bulk theory using the covariant form of the interaction.

One may wonder what the gravity dual of the Maxwell theory is. In condensed matter, there are two components of electromagnetic interaction. One is electron–electron interaction and the other is lattice–electron interaction. While the main difficulty is coming from the former, system is characterised by the latter. Working hypothesis is that the electron–electron interaction is taken care of by working in asymptotic AdS gravity. Our purpose is to include the electron–lattice interaction in this holographic scheme, which is possible for two reasons. First, in any boundary system with a conserved global $U(1)$ charge, we have a bulk Maxwell theory, which can accommodate usual electromagnetic field as a probe or an external source. It was used to build

* Corresponding author.
E-mail addresses: yseo@hanyang.ac.kr (Y. Seo), fortoe@gist.ac.kr (K.-Y. Kim), kimkyungkiu@gmail.com (K.K. Kim), sangjin.sin@gmail.com (S.-J. Sin).

[1] See however ref. [8] for semi-holographic approach based on IR AdS2 and its virtual CFT_2, which is different from ours.

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the holographic version of superconductivity mentioned above and also to calculate electric/thermal transport coefficients [10–13]. Second, we can use a relativistic theory for a non-relativistic system. The relativistic invariance highly constrains the possible form of extension of interaction. A practical way to proceed is to turn on the interaction one by one for technically simplicity. The covariant form of the interaction is either scalar or top form. The former is trivially lifted to higher dimension, e.g., $F_{\mu\nu}M^{\mu\nu}$ can be used in any dimension. Now suppose the top form of the boundary theory is F_d and the bulk theory already contains scalar operator φ and one form ω_1. Then we have essentially two choices: $\varphi\, dF_d$ and $\omega_1 \wedge F_d$ to avoid the total derivative term.

To discuss the idea in more specific context, we consider the spin–orbit interaction in 2+1 dimensional systems. It creates lots of interesting phenomena including topological insulators and Weyl semi-metal [14–18] by changing band structures, which in turn causes magneto-electric phenomena [19,20] like anomalous Hall effect. Naively, introducing the spin–orbit interaction involve fermions.[2] However, we can integrate out the massive fermions, thereby avoid dealing with fermions in our theory. Notice that in the absence of Fermi sea as in our strong coupling problem, fermions can be considered to be massive. It is well known that the fermions integrated out leave the Chern–Simons term $A \wedge F$ [21,22], which can be lifted to 4 dimension as $F \wedge F \sim E \cdot B$.[3] Since it is a total derivative by itself, we have to couple it with an appropriate scalar operator to have a non-trivial dynamical effect. In this paper, we choose it to be the kinetic energy term of the axion scalar fields χ_I. That is our interaction term is $q_\chi \sum_{I=1,2} (\partial\chi_I)^2 F \wedge F$, where χ_I was introduced to provide some disorder giving momentum dissipation [32]. Notice that this term is odd in time reversal, which is appropriate for the case where magnetisation is non-trivial.[4]

Since we want to have finite temperature, chemical potential, magnetic fields, and finite DC conductivity, the system should contain metric, gauge fields and axion scalar fields ($g_{\mu\nu}, A_\mu, \chi_I$) as the minimal ingredients in the bulk. So we have to start with the Einstein–Maxwell-axion system. We have found an exact *analytic* solution of such a non-trivially coupled system with a new interaction term, consequently yielding an explicit and analytic result for the DC conductivity using recent technology [10–12]. While the Hall effect is obviously connected to our system from the construction, the fully back reacted system shows diamagnetic response. This is because we examined metallic state at finite temperature and did not include spin degrees of freedom explicitly. Finally, we comment on the relevance of our result to experimental data. In [33], it was reported that graphite, once annealed to wash out the ferromagnetic behaviour, shows a non-linear diamagnetic response which is very similar to our analytic result. Also it turns out that our analytic conductivity formulas reproduce the experimental data on the scaling relation between the non-linear anomalous Hall coefficients and the longitudinal resistivity. I.e. the non-linear anomalous Hall coefficients interpolate between the linear and quadratic dependence on the longitudinal resistivity. Considering that we added just one interaction term, these are unexpectedly rich consequences.

The model and background solution: With motivations described above, we start from the Einstein–Maxwell-axion action with the Chern–Simons interaction

$$2\kappa^2 S = \int d^4x\sqrt{-g}\left\{R + \frac{6}{L^2} - \frac{1}{4}F^2 - \sum_{I=1,2}\frac{1}{2}(\partial\chi_I)^2\right\} - \frac{1}{16}\int q_\chi (\partial\chi_I)^2 F\wedge F + S_c\,, \tag{1}$$

where q_χ is a coupling, and $\kappa^2 = 8\pi G$ and L is the AdS radius and we set $2\kappa^2 = L = 1$. S_c is the counter term which is necessary to make the action finite. Explicit form of S_c is written in (25) at the end of this paper. The axion (χ_I) which is linear in $\{x, y\}$ direction breaks translational symmetry and hence gives an effect of momentum dissipation [32]. Instanton density coupled with the axion can generate magneto-electric property: if we add charge, non-trivial magnetisation is generated. The equations of motion are rather long so we wrote it in (26) at the end.

As ansatz to solutions, we use the following form

$$\begin{aligned} &A = a(r)dt + \frac{1}{2}H\,(xdy - ydx)\,, \\ &\chi_1 = \beta x\,, \quad \chi_2 = \beta y\,, \end{aligned} \tag{2}$$

with the metric ansatz

$$ds^2 = -U(r)dt^2 + \frac{dr^2}{U(r)} + r^2(dx^2+dy^2)\,. \tag{3}$$

From the equations of motion, we found exact solution

$$\begin{aligned} &U(r) = r^2 - \frac{\beta^2}{2} - \frac{m_0}{r} + \frac{q^2+H^2}{4r^2} + \frac{\beta^4 H^2 q_\chi^2}{20r^6} - \frac{\beta^2 H q q_\chi}{6r^4}\,, \\ &a(r) = \mu - \frac{q}{r} + \frac{\beta^2 H q_\chi}{3r^3}\,, \end{aligned} \tag{4}$$

where μ is a free parameter interpreted as the chemical potential and q and m_0 are determined by the condition $A_t(r_0) = U(r_0) = 0$ at the black hole horizon (r_0). q is the conserved $U(1)$ charge interpreted as a number density at the boundary system. m_0 turns out to be half of the energy density (9) and β is related to momentum relaxation rate

$$\begin{aligned} &q = r_0\mu + \frac{1}{3}\theta H \qquad \text{with} \quad \theta = \frac{\beta^2 q_\chi}{r_0^2}\,, \\ &m_0 = r_0^3 + \frac{r_0^2\mu^2 + H^2 - 2\beta^2 r_0^2}{4r_0} + \frac{\theta^2 H^2}{45 r_0}\,. \end{aligned} \tag{5}$$

The solution (4) reproduces the dyonic black hole solution with momentum relaxation [12] when q_χ vanishes.

Diamagnetic response: The thermodynamic potential density $\mathcal{W}$ in the boundary theory is computed by the Euclidean on-shell action S^E of (25): $S^E \equiv \mathcal{V}_2\mathcal{W}/T$, $\mathcal{V}_2 = \int dxdy$ using the solutions (2)–(3)

$$\mathcal{W} = -r_0^3 - \frac{1}{4r_0}\left(\mu^2 r_0^2 + 2\beta^2 r_0^2 - 3H^2\right) + \frac{2}{3}\mu\theta H + \frac{7}{45 r_0}\theta^2 H^2. \tag{6}$$

The system temperature T is identified with the Hawking temperature of the black hole,

$$T = \frac{3}{4\pi}r_0 - \frac{1}{16\pi r_0^3}\left((q-\theta H)^2 + H^2 + 2r_0^2\beta^2\right)\,, \tag{7}$$

[2] The Chern–Simons term is derived from a minimal interaction $\bar\psi\gamma^\mu\psi A_\mu$. If we take non-relativistic limit first, the interaction Lagrangian is $L_{int} = \vec\mu\cdot\vec B$ in the electron at rest frame, which becomes $\bar\psi\gamma^{\mu\nu}\psi F_{\mu\nu}$ in covariant form that is valid in any frame. When we include fermions explicitly, we have to take into account this issue.

[3] Previously the Chern–Simons term in the bulk and its higher dimensional analogue were extensively considered in holography to discuss the chiral effects or instability to the inhomogeneous phases [23–31].

[4] In order to handle time reversal invariant case, one can consider $q_\chi \sum_{I=1,2} d\chi_I \wedge A \wedge F \sim q_\chi \sum_{I=1,2} \chi_I F\wedge F$. One can also consider the possibility that q_χ contains an Ising spin variable ± 1 which is odd under time reversal. In this paper we focus on the time reversal breaking case to consider non-zero magnetisation.