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Studying critical string emerging from non-Abelian vortex in four dimensions

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ABSTRACT

Recently a special vortex string was found [5] in a class of soliton vortices supported in four-dimensional Yang–Mills theories that under certain conditions can become infinitely thin and can be interpreted as a critical ten-dimensional string. The appropriate bulk Yang–Mills theory has the $U(2)$ gauge group and the Fayet–Iliopoulos term. It supports semilocal non-Abelian vortices with the world-sheet theory for orientational and size moduli described by the weighted $CP(2,2)$ model. The full target space is $\mathbb{R}^4 \times Y_6$ where Y_6 is a non-compact Calabi–Yau space.

We study the above vortex string from the standpoint of string theory, focusing on the massless states in four dimensions. In the generic case all massless modes are non-normalizable, hence, no massless gravitons or vector fields are predicted in the physical spectrum. However, at the selfdual point (at strong coupling) weighted $CP(2,2)$ admits deformation of the complex structure, resulting in a single massless hypermultiplet in the bulk. We interpret it as a composite "baryon."

1. Introduction

Studies of non-Abelian vortex solitons[1] [1–4] supported by some four-dimensional Yang–Mills theories with $\mathcal{N}=2$ supersymmetry, resulted in identification of a bulk theory which under several conditions gives rise to a critical ten-dimensional string [5]. The appropriate four-dimensional Yang–Mills theory has the $U(2)$ gauge group, the Fayet–Iliopoulos term ξ and four matter hypermultiplets. Its non-Abelian sector would be conformal if it were not for the parameter ξ. It supports semilocal non-Abelian vortices with the world-sheet theory for orientational and size moduli described by the weighted $CP(2,2)$ model.[2] The target space is a six-dimensional non-compact Calabi–Yau manifold Y_6, namely, the resolved conifold. Including the translational moduli with the $\mathbb{R}^4$ target space one obtains a *bona fide* critical string, a seemingly promising discovery of [5].

In this paper we explore the spectrum of the massless modes of the above critical closed string theory. In [5] a hypothesis was formulated regarding parameters of the bulk theory and the corresponding world sheet model necessary to make the thickness of the vortex string vanish at strong coupling, which is required in order to neglect higher derivative corrections in the world sheet theory on the vortex. Here, using duality we derive an exact formula for the relation between the two-dimensional (2D) coupling β and four-dimensional (4D) gauge coupling g^2 of $\mathcal{N}=2$ SQCD. Moreover, we identify the critical point $\beta_*=0$ at which the vortex string can become infinitely thin. At this point the resolved conifold mentioned above becomes a singular conifold. As the only 4D massless mode of the string which emerges at $\beta_*=0$ we identify a single matter hypermultiplet associated with the deformation of the complex structure of the conifold. Other states arising from the ten-dimensional graviton are not dynamical in four dimensions. In particular 4D graviton and unwanted vector multiplets are absent. This is due to non-compactness of the Calabi–Yau manifold we deal with and non-normalizability of the corresponding modes.

Then we discuss the question of how the states seen in the bulk theory at weak coupling are related to what we obtain from

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[1] The non-Abelian vortex strings are understood as strings carrying non-Abelian moduli on their world sheet, in addition to translational moduli.

[2] A more accurate mathematical term is the two-dimensional sigma model with the target space $\mathcal{O}(-1)^{\oplus(2)}_{\mathbb{CP}^1}$. For brevity we will use $WCP(2,2)$.

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the string theory at strong coupling. In particular we interpret the hypermultiplet associated with the deformation of the complex structure of the conifold as a monopole–monopole baryon.

2. World sheet model

The basic bulk theory which supports the string under investigation is described in detail in [6]. Let us briefly review the model emerging on its world sheet.

The translational moduli fields (they decouple from other moduli) in the Polyakov formulation [7] are given by the action

$$S_0 = \frac{T}{2}\int d^2\sigma \sqrt{h}\, h^{\alpha\beta}\partial_\alpha x^\mu\, \partial_\beta x_\mu + \text{fermions}\,, \tag{1}$$

where σ^α ($\alpha = 1,2$) are the world-sheet coordinates, x^μ ($\mu = 1,...,4$) describe the $\mathbb{R}^4$ part of the string world sheet and $h = \det(h_{\alpha\beta})$, where $h_{\alpha\beta}$ is the world-sheet metric which is understood as an independent variable. The parameter T stands for the tension which will be discussed below.

In the bulk theory under consideration $N_f = 2N = 4$, implying that in addition to orientational zero modes of the vortex string n^P ($P = 1,2$), there are size moduli ρ^K ($K = 1,2$) [8,1,4,9–11].

The gauged formulation of the non-Abelian part is as follows [12]. One introduces the $U(1)$ charges ±1, namely $+1$ for n's and -1 for ρ's,

$$S_1 = \int d^2\sigma\sqrt{h}\left\{h^{\alpha\beta}\left(\tilde{\nabla}_\alpha \bar{n}_P\, \nabla_\beta n^P + \nabla_\alpha\bar{\rho}_K\, \tilde{\nabla}_\beta\,\rho^K\right)\right.$$
$$\left. + \frac{e^2}{2}\left(|n^P|^2 - |\rho^K|^2 - \beta\right)^2\right\} + \text{fermions}\,, \tag{2}$$

where

$$\nabla_\alpha = \partial_\alpha - iA_\alpha\,, \qquad \tilde{\nabla}_\alpha = \partial_\alpha + iA_\alpha \tag{3}$$

and A_α is an auxiliary gauge field. The limit $e^2 \to \infty$ is implied. Equation (2) represents the $WCP(2,2)$ model.[3]

The total number of real bosonic degrees of freedom in (2) is six, where we take into account constraint imposed by D-term. Moreover, one U(1) phase is gauged away. These six internal degrees of freedom are combined with four translational moduli from (1) to form a ten dimensional space needed for a superstring to be critical.

In the semiclassical approximation the coupling constant β in (2) is related to the bulk $SU(2)$ gauge coupling g^2 via

$$\beta = \frac{4\pi}{g^2}\,. \tag{4}$$

Note that the first (and the only) coefficient of the beta functions is the same for the bulk and world-sheet theories and equals to zero. This ensures that our world sheet theory is conformal invariant.

The total world-sheet action is

$$S = S_0 + S_1\,. \tag{5}$$

[3] Both the orientational and the size moduli have logarithmically divergent norms, see e.g. [9]. After an appropriate infrared regularization, logarithmically divergent norms can be absorbed into the definition of relevant two-dimensional fields [9]. In fact, the world-sheet theory on the semilocal non-Abelian string is not exactly the $WCP(N,\tilde{N})$ model [11], there are minor differences unimportant for our purposes. The actual theory is called the *zn* model. We can ignore the above differences.

3. Bulk duality vs world sheet duality

Since our vortex string is BPS saturated, the tension T in Eq. (1) is given by the exact expression

$$T = 2\pi\xi \tag{6}$$

where ξ is the Fayet–Iliopoulos parameter of the bulk theory.

As we know [13,14] the bulk theory at hand possesses a strong–weak coupling duality[4]

$$\tau \to \tau_D = -\frac{1}{\tau}\,, \qquad \tau = i\frac{4\pi}{g^2} + \frac{\theta_{4D}}{2\pi}\,. \tag{7}$$

The bulk duality implies a similar 2D duality which manifests itself in the world sheet theory as the interchange of the roles of the orientational and size moduli,

$$n^P \leftrightarrow \rho^K\,, \quad \text{or, equivalently, } \beta \to \beta_D = -\beta\,, \tag{8}$$

see Eq. (2). Equation (4) is valid only semiclassically and shows no sign of the strong–weak coupling duality (8). An obvious generalization of (4) which possess duality (8) under (7) is

$$\beta = \frac{4\pi}{g^2} - \frac{g^2}{4\pi}\,. \tag{9}$$

If $g^2 \to 16\pi^2/g^2$ then, obviously, $\beta \to -\beta$ as required by (8). The 4D selfdual point $g^2 = 4\pi$ is mapped onto $\beta_* = 0$. The selfdual point $\beta = 0$ is a critical point at which the target space $WCP(2,2)$, which is the resolved conifold, becomes a singular conifold.

It was conjectured in [5] that the non-Abelian vortex string become infinitely thin at strong coupling and can be described by the string action (5). The condition necessary for the vortex string in the bulk theory at hand to become infinitely thin is that m^2, the square of the mass of the bulk Higgsed gauge bosons, is much larger than the string tension. At weak coupling $m^2 \sim \xi\, g^2$. It is natural to assume that mass m goes to infinity at the selfdual point $\beta = 0$. This remains a hypothesis. An example of this behavior is

$$m^2 = \frac{4\pi}{|\beta|}\,\xi\,. \tag{10}$$

The expansion in derivatives of the action on the string world sheet runs in powers of ξ/m^2, implying that higher derivatives are irrelevant if $m^2 \to \infty$ [5]. Note that since the bulk theory has a vacuum manifold, there are massless states in the bulk. Most of them are not localized on the string, and therefore are irrelevant for the string study. The only localized zero modes are translational, orientational and size moduli[5] [16]. Other excitations of the string have excitation energies $\sim m^2$.

One can complexify the constant β in a standard way, by adding the topological term in the action (5) $\beta_{\text{compl}} = \beta + i\frac{\theta_{2D}}{2\pi}$, where θ_{2D} is the two-dimensional theta angle which penetrates from the bulk theory [15]. In [20] we will present a complexified version of the relation (9).

4. Bulk 4D supersymmetry from the critical non-Abelian string

The critical string discovered in [5] must lead to $\mathcal{N} = 2$ supersymmetric spectrum in our bulk four-dimensional QCD. This is *a priori* clear because our starting basic theory is $\mathcal{N} = 2$. The question is how this symmetry emerges from the string world-sheet model.

[4] Argyres et al. proved this duality for $\xi = 0$. It should allow one to study the bulk theory at strong coupling in terms of weakly coupled dual theory at $\xi \neq 0$ too.

[5] More exactly they have logarithmically divergent norms, a marginal case.