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The third order correction on Hawking radiation and entropy conservation during black hole evaporation process

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ABSTRACT

Using Parikh–Wilczek tunneling framework, we calculate the tunneling rate from a Schwarzschild black hole under the third order WKB approximation, and then obtain the expressions for emission spectrum and black hole entropy to the third order correction. The entropy contains four terms including the Bekenstein–Hawking entropy, the logarithmic term, the inverse area term, and the square of inverse area term. In addition, we analyse the correlation between sequential emissions under this approximation. It is shown that the entropy is conserved during the process of black hole evaporation, which consists with the request of quantum mechanics and implies the information is conserved during this process. We also compare the above result with that of pure thermal spectrum case, and find that the non-thermal correction played an important role.

1. Introduction

The discovery of Hawking radiation [1] greatly promoted the establishment and development of black hole thermodynamics, and it also triggered a discussion about information loss paradox for more than forty years [2]. This paradox is so important because it would provide a key ingredient in the search for a theory of quantum gravity which is expected to reconcile the disharmony among thermodynamics, relativity and quantum mechanics. Historically, L. Susskind et al. once proposed three postulates called as black hole complementarity (BHC) [3] upon a plain belief that the principle of quantum theory and a phenomenological description of a black hole should be based on, which seems to make this paradox be a little relaxation. However, from detailed searching on the underlying microphysical basis, AMPS(S) recently found that the three statements of BHC is inconsistent and proposed a firewall argument [4,5] as a conservative resolution, which arose intense attention immediately [6]. Since AdS/CFT correspondence [7] has shown that a theory for gravitation must be unitary, some works [8–15] studied this issue from quantum theory (microscopic level). Considering the consistency of physics, it should be benefit to think what clues semiclassical description of a black hole can provide to us. At a macroscopic level, Parikh once pointed out that the claim of information loss paradox rests on two pillars [16]: an exactly thermal spectrum and the validity of the no-hair theorem. Considering energy conservation and a dynamical geometry of space–time background, Parikh and Wilczek firstly proposed a tunneling model [17] for discussing Hawking radiation, and found the exact spectrum is not thermal and satisfies $\Gamma \sim \exp(\Delta S)$ [17,18], in which they have used WKB approximation. What does this non-thermal correction implies for the black hole information puzzle is an interesting problem. Parikh firstly calculated the correlation between sequential emissions and found there is no such correlation [16]. Soon after that, Arzano got the same result after taking quantum correction of black holes entropy into consideration [19]. However, using standard statistical method and distinguishing between statistical dependence or independence of two sequential emissions, B.C. Zhang firstly claimed that there exists correlation between Hawking radiations which is responsible for a process of entropy conservation, so they proposed an argument that the information is conserved [20,21]. After taking into account of the log-correction to Bekenstein–Hawking entropy, Y.X. Chen and K.N. Shao obtained a similar result [22]. Now, we will discuss Hawking radiations from a Schwarzschild black hole under the third order quantum correction and calculate the correlation between them again. It is shown that the information conservation conclusion is still true after the correction.

In Sec. 2, we applied WKB approximation at the third order correction to calculate the emission rate of a tunneling particle (S-shell) from a Schwarzschild black hole, and then the expressions for the emission spectrum and black hole entropy to the third order correction are given. In Sec. 3, we first review B.C. Zhang's

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method for calculating the correlation and the information loss paradox under lower order approximation, then apply the method to consider the case at the third order correction. In Sec. 4, we give the conclusion and compare the result with that of pure thermal spectrum. Moreover, we conclude a more general and deep result for the correlation and information conservation.

For convenience, we use geometry units ($c = G = 1$) before Eq. (22) and Planck units ($c = G = \hbar = 1$) after Eq. (23).

2. Tunneling rate at the third order correction

J.Y. Zhang first calculated the tunneling rate of Hawking radiation at the second order correction using Parikh–Wilczek tunneling framework [23,24]. Based on their works, we will make a further calculation to get a more precise tunneling rate to the third order correction.

For a Schwarzschild black hole, considering the symmetry of a particle (S-wave) created from the neighbor of horizon, the radial equation of motion is

$$\frac{1}{r^2}\frac{d}{dr}(r^2\frac{d\psi}{dr}) + \frac{2m}{\hbar^2}(E - U(r))\psi = 0. \tag{1}$$

By the substitution

$$\psi(r) = \frac{X(r)}{r}, \tag{2}$$

Eq. (1) can be rewritten as

$$\frac{d^2X}{dr^2} + [\frac{2m}{\hbar^2}(E - U(r))]X = 0. \tag{3}$$

The WKB wave function of a particle is

$$\psi(r) = \frac{X(r)}{r} = \frac{1}{r}\exp[\frac{iI(r)}{\hbar}], \tag{4}$$

where

$$I(r) = I_0(r) + (\frac{\hbar}{i})I_1(r) + (\frac{\hbar}{i})^2 I_2(r) + (\frac{\hbar}{i})^3 I_3(r) + \cdots \tag{5}$$

Putting Eqs. (4), (5) into Eq. (3), we have

$$I_0 = \pm\int^r p_r dr, \tag{6}$$

$$2I_0' I_1' + I_0'' = 0, \tag{7}$$

$$2I_0' I_2' + (I_1')^2 + I_1'' = 0, \tag{8}$$

$$2I_0' I_3' + 2I_1' I_2' + I_2'' = 0, \tag{9}$$

where $p_r = \sqrt{2m(E - U(r))}$.

Using Hamilton's equation $\dot{r} = +\frac{dH}{dp_r}|_r$, we can obtain the canonical momentum p_r in classically inaccessible region [17]. So,

$$p_r = \int_0^{p_r} dp_r' = \int \frac{dH}{\dot{r}} = -i\pi r, \tag{10}$$

we can obtain I_0', I_1', I_2' and I_3' as

$$I_0' = p_r = -i\pi r, \qquad I_0'' = -i\pi, \tag{11}$$

$$I_1' = -\frac{1}{2}\frac{I_0''}{I_0'} = -\frac{1}{2r}, \qquad I_1'' = \frac{1}{2r^2}, \tag{12}$$

$$I_2' = -\frac{1}{2I_0'}((I_1')^2 + I_1'') = -(\frac{3i}{8\pi})\frac{1}{r^3}, \qquad I_2'' = \frac{9i}{8\pi}\frac{1}{r^4}, \tag{13}$$

$$I_3' = -\frac{1}{2I_0'}(2I_1' I_2' + I_2'') = \frac{3}{4\pi^2}\frac{1}{r^5}. \tag{14}$$

Then we can write the WKB wave function to the third order approximation as

$$X(r) = \exp[\frac{i}{\hbar}(I_0 - \hbar^2 I_2) + I_1 - \hbar^2 I_3], \tag{15}$$

where

$$I_2 = \int^r I_2' dr = \frac{3i}{16\pi}\frac{1}{r^2} + C_1, \tag{16}$$

$$I_3 = \int^r I_3' dr = -\frac{3}{16\pi^2}\frac{1}{r^4} + C_2, \tag{17}$$

where C_1, C_2 are integration constants.

In order to get the tunneling rate of an emitted particle, we need to obtain the ingoing wave function and outgoing wave function. We divide the whole region by two tunneling points r_i and r_f (r_i is the initial horizon radius and r_f is the final horizon radius before and after it radiates a particle) into three regions: ingoing and reflection region I, barrier region II, and the outgoing region III. Regions I and III are classically accessible and the wave function is oscillating in these regions, but region II is classically inaccessible and the wave function is exponentially damped in this region. Considering the connections between the oscillating and the exponential solutions at $r = r_i$ and $r = r_f$ [23], the wave functions at the above three regions and the connections at those two points can be given as follows.

In region I, we have [25]

$$X_I(r) = \frac{1}{i\sqrt{v}}\exp[-\hbar^2 I_3](\exp[\frac{i}{\hbar}(\int_r^{r_i} p_r dr - \hbar^2 I_2) + \frac{i\pi}{4}] - \exp[-\frac{i}{\hbar}(\int_r^{r_i} p_r dr - \hbar^2 I_2) - \frac{i\pi}{4}]). \tag{18}$$

The connection at $r = r_i$ is

$$\frac{2}{\sqrt{v}}\exp[-\hbar^2 I_3]\sin[\frac{1}{\hbar}(\int_r^{r_i} p_r dr - \hbar^2 I_2(r)) + \frac{\pi}{4}], (r < r_i)$$
$$\rightleftharpoons \frac{1}{\sqrt{v}}\exp[-\hbar^2 I_3]\exp[-\frac{1}{\hbar}(|\int_{r_i}^{r} p_r dr| - \hbar^2 I_2)], (r > r_i) \tag{19}$$

and the connection at $r = r_f$ is

$$\frac{1}{\sqrt{v}}\exp[-\hbar^2 I_3]\exp[-\frac{1}{\hbar}(|\int_{r_f}^{r} p_r dr| - \hbar^2 I_2)], (r < r_f)$$
$$\rightleftharpoons -\frac{1}{\sqrt{v}}\exp[-\hbar^2 I_3]\exp[\frac{i}{\hbar}(\int_{r_f}^{r} p_r dr - \hbar^2 I_2) + \frac{i\pi}{4}], (r > r_f) \tag{20}$$

and the wave function in region III is

$$X_{III}(r) = -\frac{1}{\sqrt{v}}\exp[-\hbar^2 I_3]\exp[-\frac{1}{\hbar}(\mathrm{Im}I_0 - \hbar^2\mathrm{Im}I_2)] \times \exp[\frac{i}{\hbar}(\int_b^r p_r dr - \hbar^2 I_2) + \frac{i\pi}{4}], \tag{21}$$

where $v = \frac{p_r}{m}$ represents velocity of the tunneling particle.

The flux density of a wave function is