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Emergence of Euclidean dynamical symmetry as a consequence of shape phase mixing

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ABSTRACT

A hybrid model which combines γ-stable and γ-rigid collective conditions through a rigidity parameter, is used to study the critical point of the phase transition between spherical and axially symmetric shapes. The model in the equally mixed case, called $X(4)$, exhibits properties of the Euclidean symmetry in four dimensions. The spectral properties of the new model are investigated in connection to the exact symmetry. Experimental realisation of the $X(4)$ model is found in two $N = 90$ nuclei and two Pt isotopes in vicinity of experimentally observed critical point.

1. Introduction

The underlying symmetry of various aspects of the nuclear systems provides a unique insight into their structure and dynamics. Although the nucleus is essentially a many body quantum construct, it exhibits also collective features with unexpected regularities. The algebraic description of the collective phenomena based on the geometry of the system offers useful reference concepts such as dynamical symmetries (DS). Typical examples of DS in nuclear structure are those identified in the interacting boson model [1] as $U(6)$ symmetry subgroups that emerge as chains of successive algebras: $U(5)$, $O(6)$, and $SU(3)$. Each DS defines specific shapes and dynamical conditions in which the nuclei behave collectively: spherical vibrator, axially asymmetric and symmetric rotors.

As the Hamiltonian of a DS can be written in terms of Casimir operators of its group reduction chain, the corresponding energy spectrum is parameter independent and therefore serves as a reference point for realistic collective behaviour. It was found that this is also the case for the critical point symmetries (CPS) $E(5)$ [2] and $X(5)$ [3] describing the shape phase transitions between $U(5)$–$O(6)$, and respectively $U(5)$–$SU(3)$. These are actually fitting descriptions provided by similarly simple shapes of the potential surface in the geometrical Bohr model [5], which describes the quadrupole shapes using in total five variables, two associated to the nuclear shape oscillations (β and γ) and three Euler angles θ_i describing the rotational motion. Nevertheless, $E(5)$ CPS is an exact realization of the Euclidean group in five dimensions [4], while the group theoretical structure of the $X(5)$ CPS is not known. The latter employs two approximations, one related to the separation of variables and the other based on the small angles for the γ shape variable. The γ-rigid version of this model, called $X(3)$ [6], however, is exactly separable and solvable. γ-rigid conditions mean a static γ deformation and the quantum Hamiltonian associated to such a case will have a different structure as per Pauli quantization prescription [7]. Also, due to symmetry properties, rotational motion of the $X(3)$ model can be described only by two Euler angles and therefore the whole system can be described just by three variables instead of five as in the usual Bohr model. Although the resulted model seems rudimentary, its realistic character is actually supported by various experimental realisations of γ-rigid collective conditions [6,8–14].

The exact solvability is directly related to the symmetry properties of a system. Thus, the group theoretical interpretation of $X(5)$ and $X(3)$ spectral properties is of major interest. Recently [15,16], it was shown that these models are partial Euclidean DS [17,18] in the sense that a set of states satisfy exactly the associated symmetrical differential equation. In this Letter one will show that by a coherent interplay of γ-stable and γ-rigid collective conditions

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[19,20] relating the $X(5)$ and $X(3)$ models, a four dimensional model emerges which shares similar symmetry features. Additionally, a new set of states satisfying exactly the Euclidean symmetry is identified for all three $X(D)(D=1,2,3)$ models. Moreover, the rest of low lying energy states only in the four-dimensional case are approximate realisations of the corresponding Euclidean symmetry, fact which brings us closer to unveiling the symmetry group governing the CPS of the $U(5)$–$SU(3)$ shape phase transition. In completion of the present study, a theoretical and phenomenological interpretation of the new model is proposed along a handful of candidate nuclei.

2. Shape phase mixing

A combined axial symmetric γ-rigid and γ-soft nuclear system can be treated by considering the following Hamiltonian [19,20]:

$$H=\chi\hat{T}_r+(1-\chi)\hat{T}_s+V(\beta,\gamma), \tag{1}$$

where

$$\hat{T}_r=-\frac{\hbar^2}{2B}\left[\frac{1}{\beta^2}\frac{\partial}{\partial\beta}\beta^2\frac{\partial}{\partial\beta}-\frac{\mathbf{Q}^2}{3\beta^2}\right], \tag{2}$$

is the prolate γ-rigid kinetic energy operator [6], and

$$\hat{T}_s=-\frac{\hbar^2}{2B}\left[\frac{1}{\beta^4}\frac{\partial}{\partial\beta}\beta^4\frac{\partial}{\partial\beta}+\frac{1}{\beta^2\sin 3\gamma}\frac{\partial}{\partial\gamma}\sin 3\gamma\frac{\partial}{\partial\gamma}-\frac{1}{4\beta^2}\sum_{k=1}^{3}\frac{Q_k^2}{\sin^2\left(\gamma-\frac{2}{3}\pi k\right)}\right], \tag{3}$$

is the same operator corresponding to the usual five-dimensional γ-stable Bohr Hamiltonian. $\mathbf{Q}$ is the angular momentum operator from the intrinsic frame of reference with $Q_k(k=1,2,3)$ denoting the operators of its projections, while B is the mass parameter. The Ising type coupling [21,22] of different behaviours of the γ shape variable is managed by the control parameter $0\leq\chi<1$ which measures the system's γ-rigidity. The Hamiltonian (1) obviously acts in a mixed shape phase space because T_r is defined in terms of three curvilinear coordinates, while T_s in five. Therefore the integration measure of this space must be χ dependent in order to describe a coherent theory. This deformation of the shape space metric was duly explained in Ref. [20] and basically comes down to matching the quantum and classical pictures of the γ-rigid/stable coupling.

The aim of the paper is to study critical point nuclei, such that one will treat the Schrödinger equation associated to (1) as in case of the well known $X(5)$ model [3], where an approximate separation of β and γ-angular variables is achieved through a small angle approximation and an adiabatic decoupling of β and γ shape fluctuations. Assuming a factorized total wave function $\Psi(\beta,\gamma,\Omega)=\xi(\beta)\eta(\gamma)D^L_{MK}(\Omega)$ where D^L_{MK} are Wigner functions of total angular momentum L and its projections M and K on the body-fixed and respectively laboratory-fixed z axis, the associated Schrödinger equation is separated into β and γ parts [3]. The γ equation is treated as in the usual γ-stable case [3] providing a wave function indexed by the angular momentum projection K and a γ vibrational quantum number n_γ. Due to this decoupling and the additive character of the γ excitation contribution to the total energy of the system, one will concentrate in what follows only on the $K=0$ states, *i.e.* those from the ground and β excited bands. Thus, the radial-like equation for the β shape variable reads as:

$$\left[-\frac{\partial^2}{\partial\beta^2}-\frac{2(2-\chi)}{\beta}\frac{\partial}{\partial\beta}+\frac{L(L+1)}{3\beta^2}\right]\xi(\beta)+u(\beta)\xi(\beta)=\epsilon\xi(\beta), \tag{4}$$

where $\epsilon=\frac{2B}{\hbar^2}E$ and $u(\beta)=\frac{2B}{\hbar^2}V(\beta)$ are reduced energy and β potential. In accordance to $X(5)$ [3] and $X(3)$ [6] critical point solutions, an anharmonic behaviour is considered here for the potential, reflected into a square well shape:

$$u(\beta)=\begin{cases}0, & \beta\leqslant\beta_W,\\ \infty, & \beta>\beta_W,\end{cases} \tag{5}$$

where β_W indicates the position of the infinite wall. With this, equation (4) can be brought to a Bessel differential equation by the change of variable $\xi(\beta)=\beta^{\chi-\frac{3}{2}}f(\beta)$:

$$\left[\frac{\partial^2}{\partial\beta^2}+\frac{1}{\beta}\frac{\partial}{\partial\beta}+\left(k^2-\frac{\nu^2}{\beta^2}\right)\right]f(\beta)=0, \tag{6}$$

where

$$\nu=\sqrt{\frac{L(L+1)}{3}+\left(\frac{3}{2}-\chi\right)^2}. \tag{7}$$

The boundary condition $f(\beta_W)=0$ gives the β energy spectrum in terms of the s-th zero $x_{s,\nu}$ of the Bessel function $J_\nu(x_{s,\nu}\beta/\beta_W)$ [23]. The order of the Bessel function's zero is related to the β vibration quantum number by $n_\beta=s-1$. Correspondingly, the β variable normalized wave function is then given as:

$$\xi_{L,n_\beta}(\beta)=N_{n_\beta,\nu}\beta^{\chi-\frac{3}{2}}J_\nu(x_{n_\beta+1,\nu}\beta/\beta_W), \tag{8}$$

where $N_{n_\beta,\nu}$ is the normalization constant obtained from the condition

$$\int_0^{\beta_W}\left[\xi_{L,n_\beta}(\beta)\right]^2\beta^{4-2\chi}d\beta=1. \tag{9}$$

Note the modified integration measure which accounts for the shape phase mixing [20]. Another example of deformed shape phase space is the recently proposed collective solution with an energy dependent potential [24].

Finally, the total solution of the Hamiltonian (1) is given by the normalized and symmetrized product of angular, β and γ wave functions [3,25]:

$$\Psi_{LMKn_\beta n_\gamma}(\beta,\gamma,\Omega)=\xi_{L,n_\beta}(\beta)\eta_{n_\gamma,|K|}(\gamma)\times\sqrt{\frac{2L+1}{16\pi^2(1+\delta_{K,0})}}\left[D^L_{MK}(\Omega)+(-)^LD^L_{M-K}(\Omega)\right]. \tag{10}$$

Transition rates can then be calculated by employing the general expression for the quadrupole transition operator,

$$T^{(E2)}_\mu=t\beta\left[D^2_{\mu0}\cos\gamma+\frac{1}{\sqrt{2}}\left(D^2_{\mu2}+D^2_{\mu-2}\right)\sin\gamma\right], \tag{11}$$

where t is a scaling factor. Taking into account the small angle approximation ($\cos\gamma\approx1$) appropriate for γ-stable solutions, the $\Delta K=0$ transitions relating the ground and β excited states relevant for the present study can be given in a factorized form as [25,26]:

$$B(E2;Ln_\beta\to L'n'_\beta)=\frac{5t^2}{16\pi}\left(C^{L2L'}_{000}B^{Ln_\beta}_{L'n'_\beta}\right)^2. \tag{12}$$

C is the Clebsch–Gordan coefficient dictating the angular momentum selection rules, while B is defined as: