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New family of Maxwell like algebras

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ABSTRACT

We introduce an alternative way of closing Maxwell like algebras. We show, through a suitable change of basis, that resulting algebras are given by the direct sums of the AdS and the Maxwell algebras already known in the literature. Casting the result into the S-expansion method framework ensures the straightaway construction of the gravity theories based on a found enlargement.

1. Introduction

While contractions and (corresponding to an inverse process) deformations share the property of preserving the dimension of the Lie algebra, there are some procedures that allow us to find algebras with a greater number of generators, even in a way that the original algebra is not necessarily included as a subalgebra. An example of such algebraic enlargement for the Poincaré case was found in Refs. [1,2]. The Maxwell algebra presented there was used to describe the symmetries of quantum fields in the Minkowski spacetime in a presence of the constant electromagnetic field strength tensor. Its semisimple version appeared in Refs. [3,4] and represents the direct sum of the AdS and Lorentz algebras.

Recently, the both mentioned examples, along with their supersymmetric extensions, have been further extended by using generalized contractions known as Lie algebra expansion methods [5,6]. Together with a later reformulation in terms of the abelian semigroups called the S-expansion [7], these expansion methods proved to be a powerful tool generating the new theories of gravity [8–14] and supergravity [15–18]. Besides further studying the new supersymmetric schemes [19,20], the subject finds also other applications. In Ref. [21] the cosmological constant term in four dimensions arises from the Maxwell algebra. The gauge fields related to the new generators might be useful in inflation theories driven by the vector fields [22] coupled to gravity in a suitable way. Introduction of new fields and invariant tensors affects the final form of the Lagrangians, which was particularly exploited in Refs. [8,9] to establish a relation between General Relativity (GR) and Chern–Simons (CS) gravity in odd dimensions. The same has been achieved for even dimensions to relate Born–Infeld (BI) gravity with GR [23]. Other applications in the context of Bianchi algebras and a study of properties of the S-expansion procedure with general semigroups have also been analyzed in Refs. [24,25].

In this paper we introduce another family of Maxwell like algebras. Although its existence could be understood in the S-expansion framework, we will not write it explicitly from the very beginning. We will start by including new generators $\{Z_{ab}, R_a, ...\}$ (with $a, b = 1, \dots, d$) to the Lorentz and translational generators, adopting most of the conventions and general setting presented in Ref. [14]. We present a new scheme for closing the enlarged algebras in a way different to already known Maxwell families. By generalizing the change of basis from Ref. [3], we discover that the newly obtained algebras, denoted as $\mathfrak{D}_m$, can be seen as the direct sum of the AdS and the $\mathfrak{B}_{m-2}$ algebras obtained by the expansion method [9]. In addition, $\mathfrak{D}_m$ algebras lead to $\mathfrak{B}_m$ under the Inönü–Wigner contraction. Finally, we explicitly incorporate these results within the S-expansion context and discuss the gravity actions in odd and even dimensions.

2. Maxwell algebras

The S-expansion procedure allows us to obtain two separate types of algebras, denoted in the literature as $\mathfrak{B}_m$ and $AdS\mathcal{L}_m$ [9,13,14]. Both can be related with each other by the Inönü–Wigner contraction. Integer index $m > 2$ labels different representatives, where standard generators of the Lorentz transformations J_{ab} and translations P_a become equipped with another set (or sets) of the new generators Z_{ab} and R_a. Value $(m-1)$ might be used to indi-

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cate the total number of different generators $\{J_{ab}, P_a, Z_{ab} = Z^{(1)}_{ab}, R_a = R^{(1)}_a, Z^{(2)}_{ab}, R^{(2)}_a, \ldots\}$.

Our starting point is the Poincaré and AdS algebras, which can be identified with $\mathfrak{B}_3$ and $AdS\mathcal{L}_3$, respectively. Including the Z_{ab} generator leads to $\mathfrak{B}_4$, which represents the Maxwell algebra [1,2]. It can be seen as a contraction of the $AdS\mathcal{L}_4$ algebra [3,14], in which the used notation resembles the fact that it combines the AdS with an additional Lorentz algebra. That algebra (along with its supersymmetric extension) originally appeared in [3,4] and was described as tensorial semi-simple enlargement of the Poincaré algebra. This was followed by a discussion of its deformations [26], and later reappeared in yet another form of Maxwellian deformation of the AdS algebra [27] with $[P_a, P_b] = (J_{ab} - Z_{ab})$. This last form reflects just the change of basis with different decomposition of the generators $\{J_{ab}, P_a, Z_{ab}\}$ forming the direct sum of $\mathfrak{so}(d-1,2) \oplus \mathfrak{so}(d-1,1)$ either out of AdS $\{Z_{ab}, P_a\} \oplus$ Lorentz $\{(J-Z)_{ab}\}$ or AdS $\{(J-Z)_{ab}, P_a\} \oplus$ Lorentz $\{Z_{ab}\}$ generators. It would be interesting to somehow relate this with the symmetry of fields, like was done in Ref. [1], but now in the AdS spacetime with the constant electromagnetic field.

Name, originally used for $\mathfrak{B}_4$, was extended to describe further generalizations for any index m, and also to take into account the semi-simple Poincaré enlargement and its generalizations. Finally, they all could be referred to as Maxwell type algebras [13] or generalized Maxwell algebras [28]. Since $AdS\mathcal{L}_3$ coincides with the AdS we find using label $AdS\mathcal{L}_m$ and the name *generalized AdS–Lorentz* from Ref. [14] a little bit misleading. Indeed, only in one case ($m = 4$) we can talk about the direct sum of AdS and Lorentz, while for $m > 4$ the AdS algebra is no longer present as a subalgebra. Therefore, throughout the paper we propose changing the label $AdS\mathcal{L}_m$ into $\mathfrak{C}_m$, which will better fit the scheme presented in this work.

In the case of $\mathfrak{B}_5$ and $\mathfrak{C}_5$ (formerly in [14] called as $AdS\mathcal{L}_5$) we can write their common part of the commutation relations as

$$
\begin{aligned}
[P_a, P_b] &= Z_{ab}\,,\\
[J_{ab}, P_c] &= \eta_{bc} P_a - \eta_{ac} P_b\,,\\
\left[J_{ab}, J_{cd}\right] &= \eta_{bc} J_{ad} + \eta_{ad} J_{bc} - \eta_{ac} J_{bd} - \eta_{bd} J_{ac}\,,\\
\left[J_{ab}, Z_{cd}\right] &= \eta_{bc} Z_{ad} + \eta_{ad} Z_{bc} - \eta_{ac} Z_{bd} - \eta_{bd} Z_{ac}\,,\\
[Z_{ab}, P_c] &= \eta_{bc} R_a - \eta_{ac} R_b\,,\\
[J_{ab}, R_c] &= \eta_{bc} R_a - \eta_{ac} R_b\,.
\end{aligned}
\tag{1}
$$

When the algebra closes by satisfying

$$
\begin{aligned}
[R_a, R_b] &= Z_{ab}\,,\\
[Z_{ab}, R_c] &= \eta_{bc} P_a - \eta_{ac} P_b\,,\\
\left[Z_{ab}, Z_{cd}\right] &= \eta_{bc} J_{ad} + \eta_{ad} J_{bc} - \eta_{ac} J_{bd} - \eta_{bd} J_{ac}\,,\\
[R_a, P_b] &= J_{ab}\,,
\end{aligned}
\tag{2}
$$

we obtain $\mathfrak{C}_5$. After the following rescaling

$$
P_a \to \mu P_a\,, \qquad Z_{ab} \to \mu^2 Z_{ab}\,, \qquad \text{and} \qquad R_a \to \mu^3 R_a\,, \tag{3}
$$

the Inönü–Wigner (IW) contraction [29] of the $\mathfrak{C}_5$ algebra in the limit of dimensionless parameter $\mu \to \infty$ shares exactly the form of common part (1), whereas the remaining commutators become

$$
\begin{aligned}
[R_a, R_b] &= 0\,,\\
[Z_{ab}, R_c] &= 0\,,\\
[Z_{ab}, Z_{cd}] &= 0\,,\\
[R_a, P_b] &= 0\,.
\end{aligned}
\tag{4}
$$

It describes $\mathfrak{B}_5$, whose applications in Refs. [8,9] were already mentioned in the Introduction.

As we will see in the next section the separation on the two subsets of the commutation relation is crucial to find a new algebra.

3. Direct Maxwell algebras

Intriguingly, there is one more way to close the subset of commutators listed in (1), which is given by

$$
\begin{aligned}
[R_a, R_b] &= Z_{ab}\,,\\
[Z_{ab}, R_c] &= \eta_{bc} R_a - \eta_{ac} R_b\,,\\
\left[Z_{ab}, Z_{cd}\right] &= \eta_{bc} Z_{ad} + \eta_{ad} Z_{bc} - \eta_{ac} Z_{bd} - \eta_{bd} Z_{ac}\,,\\
[R_a, P_b] &= Z_{ab}\,.
\end{aligned}
\tag{5}
$$

The result, surprisingly, can be seen as a direct sum of two subalgebras. As we will see, this example opens the whole new family of algebras where, in contrast to $\mathfrak{C}_{m>4}$, the AdS subalgebra is always present. To show this, we introduce a generalization of the change of basis presented in Ref. [3], which now applies also to the "translational" generator. With the group indices specified as $a, b = 1, \ldots, d$ we define two sets of generators

$$
L_{IJ} = \begin{cases} L_{ab} = Z_{ab}\,, \\ L_{a(D+1)} = R_a\,, \end{cases} \qquad \text{and} \qquad N_{IJ} = \begin{cases} N_{ab} = (J_{ab} - Z_{ab})\,, \\ N_a = (P_a - R_a)\,, \end{cases} \tag{6}
$$

satisfying the AdS

$$
[L_{IJ}, L_{KL}] = \eta_{JK} L_{IL} + \eta_{IL} L_{JK} - \eta_{IK} L_{JL} - \eta_{JL} L_{IK}\,, \tag{7}
$$

and the Poincaré algebra

$$
\begin{aligned}
\left[N_{ab}, N_{cd}\right] &= \eta_{bc} N_{ad} + \eta_{ad} N_{bc} - \eta_{ac} N_{bd} - \eta_{bd} N_{ac}\,,\\
[N_{ab}, N_c] &= \eta_{bc} N_a - \eta_{ac} N_b\,, \qquad [N_a, N_b] = 0\,.
\end{aligned}
\tag{8}
$$

It is straightforward to check that

$$
[L_{IJ}, N_{KL}] = 0, \tag{9}
$$

therefore, they form the direct sum of $\mathfrak{so}(d-1,2) \oplus \mathfrak{iso}(d-1,1)$. From now on we will denote this algebra as $\mathfrak{D}_5$, where the used letter emphasizes the direct character of the found structure.

Similarly, for one more generator, $Z^{(2)}_{ab} = \hat{Z}_{ab}$, added to $\{J_{ab}, P_a, Z_{ab}, R_a\}$, the new algebra $\mathfrak{D}_6$ will share with the $\mathfrak{B}_6$ and $\mathfrak{C}_6$ the same subset of commutators

$$
\begin{aligned}
[P_a, P_b] &= Z_{ab}\,,\\
[J_{ab}, P_c] &= \eta_{bc} P_a - \eta_{ac} P_b\,,\\
\left[J_{ab}, J_{cd}\right] &= \eta_{bc} J_{ad} + \eta_{ad} J_{bc} - \eta_{ac} J_{bd} - \eta_{bd} J_{ac}\,,\\
\left[J_{ab}, Z_{cd}\right] &= \eta_{bc} Z_{ad} + \eta_{ad} Z_{bc} - \eta_{ac} Z_{bd} - \eta_{bd} Z_{ac}\,,\\
[Z_{ab}, P_c] &= \eta_{bc} R_a - \eta_{ac} R_b\,,\\
[J_{ab}, R_c] &= \eta_{bc} R_a - \eta_{ac} R_b\,,\\
\left[Z_{ab}, Z_{cd}\right] &= \eta_{bc} \hat{Z}_{ad} + \eta_{ad} \hat{Z}_{bc} - \eta_{ac} \hat{Z}_{bd} - \eta_{bd} \hat{Z}_{ac}\,,\\
\left[J_{ab}, \hat{Z}_{cd}\right] &= \eta_{bc} \hat{Z}_{ad} + \eta_{ad} \hat{Z}_{bc} - \eta_{db} \hat{Z}_{bd} - \eta_{bd} \hat{Z}_{ac}\,,\\
[P_a, Z_b] &= \hat{Z}_{ab}\,.
\end{aligned}
\tag{10}
$$

Additional rules reproducing $\mathfrak{C}_6$ are provided through