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# Heavy fermions and two loop corrections to $(g-2)_\mu$

Xiu-Yi Yang [a,b,*], Tai-Fu Feng [a,b]

[a] Department of Physics, Dalian University of Technology, Dalian 116024, China
[b] Center for High Energy Physics, Peking University, Beijing 100871, China

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ABSTRACT

Applying effective Lagrangian method and on-shell scheme, we analyze the electroweak corrections to anomalous dipole moments of lepton from some special two loop diagrams in which a closed heavy fermion loop is attached to the virtual gauge bosons or Higgs fields. As the masses of virtual fermions in inner loop are much heavier than the electroweak scale, we verify the final results satisfying the decoupling theorem explicitly if the interactions among Higgs and heavy fermions do not contain the nondecoupling couplings. At the decoupling limit, we also present the leading corrections to lepton anomalous dipole moments from those two loop diagrams in some popular extensions of the standard model, such as the fourth generation, supersymmetry, and the littlest Higgs with T-parity.



The current experimental average of $(g-2)_\mu$ is [1]

$$a_\mu^{\exp} = 11\,659\,208 \pm 6 \times 10^{-10}. \tag{1}$$

Depending on which evaluation of hadronic vacuum polarization is chosen, the differences between the SM predictions and experimental result lie in the range $1.3\sigma$–$3.8\sigma$ [2,3].

For the electroweak corrections, the one loop corrections from new physics sector are generally suppressed by $\Lambda_{\rm EW}^2/\Lambda_{\rm NP}^2$. Here $\Lambda_{\rm EW}$ denotes the electroweak energy scale, and $\Lambda_{\rm NP}$ denotes the energy scale of new physics. Comparing with the analysis at one loop level, the two loop analysis is more complicated and less advanced [4–8].

We have presented all techniques in detail how to get the corrections to anomalous dipole moments of lepton from some special diagrams in which a closed heavy fermion loop is attached to the virtual electroweak gauge or Higgs fields [9], and investigated the corrections to lepton anomalous dipole moments from chargino/neutralino sector through those two loop diagrams in the split supersymmetry [10] and CP violating minimal supersymmetric extension of the SM (MSSM) [11], respectively. Within the framework of MSSM, our predictions on $(g-2)_\mu$ agree with the corresponding results in Ref. [5] very well [9].

In this Letter, we mainly discuss the asymptotic behavior of the corrections from those two loop diagrams in the decoupling limit. Using those asymptotic formulae at the decoupling limit, one can evaluate roughly the magnitude of the corrections to lepton magnetic dipole moments (MDMs) from those two loop diagrams in some popular extensions of the standard model. Here, the effective Lagrangian method [12] and on-shell renormalization scheme [13] are applied to get $(g-2)_\mu$ in our calculation. We adopt the naive dimensional regularization with the anticommuting $\gamma_5$ scheme and the nonlinear $R_\xi$ gauge [14] with $\xi = 1$.

In order to get the amplitude of the diagrams in Fig. 1(a), one can write the renormalizable interaction among the charged electroweak gauge boson $W^\pm$ and the heavy fermions $F_{\alpha,\beta}$ in a more universal form as

$$\mathcal{L}_{WFF} = \frac{e}{s_{\rm w}} W^{-,\mu} \bar{F}_\alpha \gamma_\mu \left(\zeta^L_{\alpha\beta}\omega_- + \zeta^R_{\alpha\beta}\omega_+\right) F_\beta + \text{h.c.}, \tag{2}$$

* Corresponding author at: Department of Physics, Dalian University of Technology, Dalian 116024, China.
*E-mail address:* yxyruxi@163.com (X.-Y. Yang).

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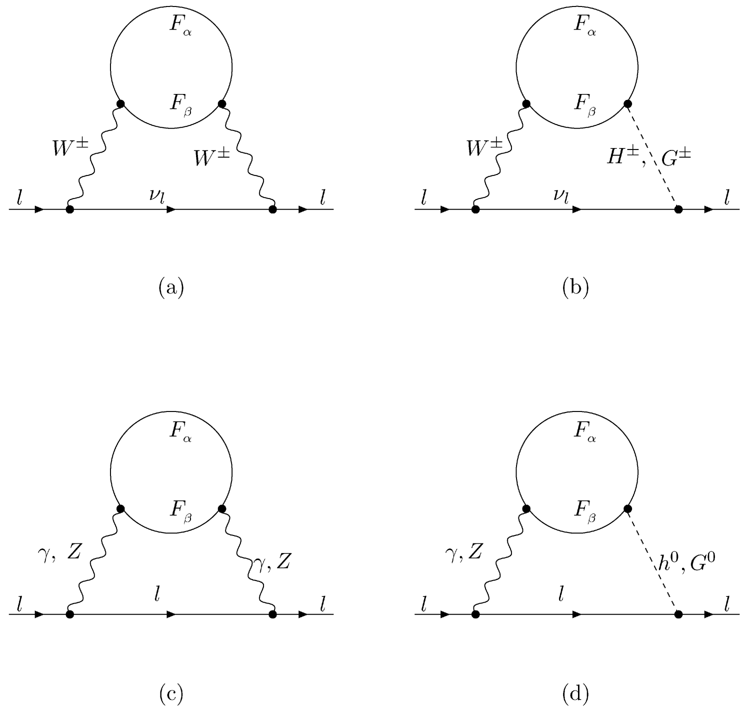


**Fig. 1.** The relating two-loop diagrams in which a closed heavy fermion loop is attached to the virtual gauge bosons or Higgs fields. Where the photon can emit in all possible ways.

where the concrete expressions of $\zeta^{L,R}_{\alpha\beta}$ depend on the models employed in our calculation. The conservation of electric charge requires $Q_\beta - Q_\alpha = 1$, where $Q_{\alpha,\beta}$ denote the electric charges of the heavy fermions $F_{\alpha,\beta}$, respectively.

The charged gauge boson self energy composed of a closed heavy fermion loop induces the ultraviolet divergence in the Wilson coefficients of effective Lagrangian, the unrenormalized $W^\pm$ self energy is generally expressed as

$$\Sigma^{\rm w}_{\mu\nu}(p, \Lambda_{\rm RE}) = \Lambda^2 A^{\rm w}_0 g_{\mu\nu} + \left(A^{\rm w}_1 + \frac{p^2}{\Lambda^2} A^{\rm w}_2 + \cdots\right)\left(p^2 g_{\mu\nu} - p_\mu p_\nu\right) + \left(B^{\rm w}_1 + \frac{p^2}{\Lambda^2} B^{\rm w}_2 + \cdots\right) p_\mu p_\nu, \tag{3}$$

where the form factors $A^{\rm w}_{0,1,2}$ and $B^{\rm w}_{1,2}$ only depend on the virtual field masses and renormalization scale. We derive the counter terms for the $W^\pm$ self energy in on-shell scheme as

$$\delta Z^{\rm os}_{\rm w}(m_{\rm w}) = A^{\rm w}_1 + \frac{m^2_{\rm w}}{\Lambda^2} A^{\rm w}_2 = A^{\rm w}_1 + x_{\rm w} A^{\rm w}_2, \qquad \delta m^{2,{\rm os}}_{\rm w}(m_{\rm w}) = A^{\rm w}_0 \Lambda^2 - m^2_{\rm w} \delta Z^{\rm os}_{\rm w}. \tag{4}$$

We should derive the counter term for the vertex $\gamma W^+ W^-$ here since the corresponding coupling is not zero at tree level. In the nonlinear $R_\xi$ gauge with $\xi = 1$, the counter term for the vertex $\gamma W^+ W^-$ is

$$i\delta C_{\gamma W^+ W^-} = ie \cdot \delta Z_{\rm w}(\Lambda_{\rm RE})\left[g_{\mu\nu}(k_1 - k_2)_\rho + g_{\nu\rho}(k_2 - k_3)_\mu + g_{\rho\mu}(k_3 - k_1)_\nu\right], \tag{5}$$

where $k_i$ $(i = 1, 2, 3)$ denote the incoming momenta of $W^\pm$ and photon, and $\mu, \nu, \rho$ denote the corresponding Lorentz indices, respectively.

Under our approximation, the resulted lepton MDM is formulated as

$$a^{\rm ww}_{l,F} = \frac{G_F \alpha_e m^2_l}{2\sqrt{2}\pi^3 s^2_{\rm w}} x_{\rm w} \Big\{\Big(\big|\zeta^L_{\alpha\beta}\big|^2 + \big|\zeta^R_{\alpha\beta}\big|^2\Big) T_1(x_{\rm w}, x_{F_\alpha}, x_{F_\beta}) + \Big(\big|\zeta^L_{\alpha\beta}\big|^2 - \big|\zeta^R_{\alpha\beta}\big|^2\Big) T_2(x_{\rm w}, x_{F_\alpha}, x_{F_\beta}) + 2(x_{F_\alpha} x_{F_\beta})^{1/2} \Re\big(\zeta^{R*}_{\alpha\beta} \zeta^L_{\alpha\beta}\big) T_3(x_{\rm w}, x_{F_\alpha}, x_{F_\beta})\Big\}, \tag{6}$$

which only depend on the masses of virtual fields. where $G_F = 1.16639 \times 10^{-5}$ GeV$^{-2}$ is the 4-fermion coupling, and $\alpha_e = e^2/4\pi$. Note that the above result does not depend on the concrete choice of energy scale $\Lambda$, and the concrete expressions of $T_i(x, y, z)$, $\varrho_{i,j}(x, y)$ $(i, j = 1, 2, \ldots)$ can be found in Ref. [9], which are expressed in linear combinations of one- and two-loop vacuum integrals.

Under the assumption $m_F = m_{F_\alpha} = m_{F_\beta} \gg m_{\rm w}$, we get the leading contributions contained in the asymptotic form of Eq. (6) as:

$$a^{\rm ww}_{l,F} \approx \frac{G_F \alpha_e m^2_l}{48\sqrt{2}\pi^3 s^2_{\rm w}} \Big\{(18 Q_\beta - 13)\Big(\big|\zeta^L_{\alpha\beta}\big|^2 + \big|\zeta^R_{\alpha\beta}\big|^2\Big) + 3(Q_\beta - 3)\Big(\big|\zeta^L_{\alpha\beta}\big|^2 - \big|\zeta^R_{\alpha\beta}\big|^2\Big) + 11\Re\big(\zeta^{R*}_{\alpha\beta} \zeta^L_{\alpha\beta}\big)\Big\} + \cdots, \tag{7}$$

where ellipses represent those relatively unimportant corrections.

Similarly, the renormalizable interaction among the electroweak charged Goldstone/Higgs $G^\pm(H^\pm)$ and the heavy fermions $F_{\alpha,\beta}$ can be expressed in a more universal form as

$$\mathcal{L}_{S^\pm FF} = \frac{e}{s_{\rm w}}\left[G^- \bar{F}_\alpha \left(\mathcal{G}^{c,L}_{\alpha\beta} \omega_- + \mathcal{G}^{c,R}_{\alpha\beta} \omega_+\right) F_\beta + H^- \bar{F}_\alpha \left(\mathcal{H}^{c,L}_{\alpha\beta} \omega_- + \mathcal{H}^{c,R}_{\alpha\beta} \omega_+\right) F_\beta\right] + {\rm h.c.}, \tag{8}$$