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Dark energy, inflation and the cosmic coincidence problem

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Abstract

We show that holographic dark energy could explain why the current dark energy density is so small, if there was an inflation with a sufficient expansion in the early universe. It is also suggested that an inflation with the number of e-folds $N \simeq 65$ may solve the cosmic coincidence problem in this context. Assuming the inflation and the power-law acceleration phase today we obtain approximate formulas for the event horizon size of the universe and dark energy density as functions of time. A simple numerical study exploiting the formula well reproduces the observed evolution of dark energy. This nontrivial match between the theory and the observational data supports both inflation and holographic dark energy models.

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The type Ia supernova (SN Ia) observations [1,2] strongly suggest that the current universe is in an accelerating phase, which can be explained by dark energy (a generalization of the cosmological constant) having pressure p_Λ and density ρ_Λ such that $\omega_\Lambda \equiv p_\Lambda/\rho_\Lambda < -1/3$. There are various dark energy models rely on exotic materials such as quintessence [3,4], k-essence [5,6], phantom [7], and Chaplygin gas [8,9]. Being one of the most important unsolved puzzles in modern physics, the cosmological constant problem consists of three sub-problems; why the cosmological constant is so small, nonzero, and comparable to the critical density at the present.

In this Letter we show that, in the holographic dark energy model, an inflation with a sufficient expansion explain why the current dark energy density is so small. We also suggest that the last problem, the cosmic coincidence problem, could be solved, if there was an inflation with a specific expansion. Note that, in many other dark energy models, it is not easy to explain the current ratio of dark energy density to matter energy density, because usually dark energy density and matter energy density reduce at different rates [10] for a long cosmological time scale.

It is well known [11] that a simple combination of the reduced Planck mass $M_P = m_P/\sqrt{8\pi}$ and the Hubble parameter $H = H_0 \sim 10^{-33}$ eV, gives a value $\rho_\Lambda \simeq M_P^2 H_0^2$ comparable to the observed dark energy density $\sim 10^{-10}\ \mathrm{eV}^4$ [2]. This interesting coincidence, on one hand, is of the cosmic coincidence problem and, on the other hand, motivated holographic dark energy models. The holographic dark energy models are based on the holographic principle proposed by 't Hooft and Susskind [12–14], claiming that all of the information in a volume can be described by the physics at the boundary of the volume. With the base on the principle, Cohen et al. [15] proposed a relation between an UV cutoff (a) and an IR cutoff (L) by considering that the total energy in a region of size L cannot be larger than the mass of a black hole of that size. Saturating the bound, one can obtain

$$\rho_\Lambda = \frac{3d^2}{L^2 a^2}, \tag{1}$$

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where d is a constant. Hsu [16] pointed out that for $L = H^{-1}$, the holographic dark energy behaves like matter rather than dark energy. Many attempts [17–22] have been made to overcome this IR cutoff problem, for example, by using non-minimal coupling to a scalar field [20,21] or an interaction between dark energy and dark matter [22–26]. Li [27,28] suggested that an ansatz for the holographic dark energy density

$$\rho_\Lambda = \frac{3d^2 M_P^2}{R_h^2}, \tag{2}$$

would give a correct accelerating universe, where the future event horizon (R_h) is used instead of the Hubble horizon as the IR cutoff L.

To solve the coincidence problem many attempts have been done [24,29–34]. An interaction of dark matter [35] with dark energy was introduced in [23,36,37]. In [32] inflation at the GUT scale with the minimal number of e-folds $N \simeq 60$ was suggested as a solution. In this Letter we suggest a solution similar to the later. One motivation to study the cosmic coincidence problem in the context of inflationary cosmology is that if there was no inflation, there could be no 'now' ($t_0 = 1.37 \times 10^{10}$ years) for the 'why now' question. According to astronomical observations and cosmological theory there are at least two inflationary periods in the history of the universe. As is well known, the first inflation at the early universe with $N > 60$ is need to solve the problems of the standard big-bang cosmology. This inflation is often assumed to be related to vacuum energy of a scalar field (inflaton). The second inflation (re-inflation) is a period of an accelerated expansion today due to dark energy. (Usually, the first inflation is related to a phase transition of the inflaton and has a different origin from that of the re-inflation due to dark energy. In this Letter we assume this case.) Thus, we assume that in the universe there are the inflaton, holographic dark energy, radiation and matter (mostly, cold dark matter). We also assume that after reheating inflaton energy decays to radiation perfectly. During the first inflation holographic dark energy is diluted exponentially. In this work we suggest that if there is holographic dark energy in the universe, the first inflation with $N \simeq 65$ leads to onset of the second inflation at the time $t_a = O(10^9)$ years as observed, and, hence, the inflation solves the cosmic coincidence problem in the context of holographic dark energy.

In this Letter we consider the flat ($k = 0$) Friedmann universe which is favored by observations [38] and described by the metric

$$ds^2 = -dt^2 + R^2(t)\, d\Omega^2, \tag{3}$$

where $R(t)$ is the scale factor. In the holographic dark energy model a typical length scale of the system with the horizon is given by the future event horizon

$$R_h \equiv R(t) \int_t^\infty \frac{dR(t')}{H(t')R(t')^2} = R(t) \int_t^\infty \frac{dt'}{R(t')}, \tag{4}$$

which is a key quantity. It is a subtle task to obtain an explicit form for $R_h(t)$, because $R_h(t)$ depends on the whole history of the universe after t. To tackle this problem we divide the history of the universe into two phases; the inflation (phase 1) is followed by phase 2 which are consecutive radiation dominated era (RDE; $R(t) \propto t^{1/2}$) and dark energy dominated power-law accelerating era (DDE; $R(t) \propto t^n, n > 1$), respectively. For simplicity, we ignore the matter dominated era (MDE) as often done in an order of magnitude estimate in cosmology. (In Appendix A, we perform a similar calculation with MDE. The main results are similar.)

1) inflation phase ($t_i \leqslant t < t_f$).

The inflation starts at $t = t_i$ and ends at t_f. The scale factor evolves in this phase as follows

$$R(t) = R_i e^{H_i(t-t_i)}, \tag{5}$$

where R_i is the initial scale factor at $t = t_i$ and $H_i = M_i^2/(\sqrt{3}M_P)$ is the Hubble parameter with the energy scale M_i of the inflation. Hence, the number of e-folds of expansion $N \equiv H_i(t_f - t_i)$.

2) power-law expansion phase ($t_f \leqslant t < \infty$).

This phase consists of RDE ($t_f \leqslant t < t_a$) followed by DDE ($t_a \leqslant t < \infty$). The universe starts to accelerate at an inflection point $t = t_a$, i.e., $\ddot{R}(t_a) = 0$. We assume that the scale factor evolves in this phase as

$$R(t) = R_i e^N \left(\frac{t}{t_f}\right)^{\frac{1}{2}} \left(\frac{1 + \alpha(\frac{t}{t_f})^{1/2}}{1+\alpha}\right)^{2n}, \tag{6}$$

where $\alpha \simeq (t_f/t_a)^{1/2}$ is a constant. The scale factor $R(t)$ grows as $t^{\frac{1}{2}}$ during the RDE and as $t^{n+\frac{1}{2}}$ during the DDE later. $R(t)$ of this form gives a smooth transition from RDE to DDE. Note that $R(t)$ for each era is well known and can be derived from the Friedmann equation depending on the dominant energy source. The power-law acceleration is a generic feature of DDE if $d > 1$. (Alternatively, one can divide this phase into RDE and DDE and choose the scale factor as $R(t) \propto (t/t_f)^{1/2}$ and $R(t) \propto (t/t_a)^n$ for RDE and DDE, respectively. This choice gives almost the same results except for a slightly decreasing R_h as $t \to t_a$. Thus, we can use the specific form in Eq. (6) without loss of generality.) Since observational data favor $d \simeq 1$ [39,40] and ω_Λ close to