Available online at www.sciencedirect.com
ScienceDirect
Physics Letters B 661 (2008) 88–94
PHYSICS LETTERS B
www.elsevier.com/locate/physletb

The pressure of QED from the two-loop 2PI effective action

Sz. Borsányi [a], U. Reinosa [b,*]

[a] *Department of Physics and Astronomy, University of Sussex, Brighton, East Sussex BN1 9QH, United Kingdom*
[b] *Institut für Theoretische Physik, Universität Heidelberg, Philosophenweg 16, 69120 Heidelberg, Germany*

Received 2 October 2007; received in revised form 5 February 2008; accepted 6 February 2008

Available online 8 February 2008

Editor: A. Ringwald

Abstract

We compute the pressure of hot quantum electrodynamics from the two-loop truncation of the 2PI effective action. Since the 2PI resummation guarantees gauge-fixing independence only up to the order of the truncation, our result for the pressure presents a gauge-dependent contribution of $\mathcal{O}(e^4)$. We numerically characterize the credibility of this gauge-dependent calculation and find that the uncertainty due to gauge parameter dependence is under control for $\xi \lesssim 1$. Our calculation also suggests that the choice of Landau gauge may minimize gauge-dependent effects.

The diagrammatic approach to relativistic quantum field theories heavily relies on the convergence properties of the used expansion scheme. Among the various resummation schemes which have been invented to cure the poor performance of the perturbative expansion [1] in various situations of interest, the loop expansion of the two-particle-irreducible (2PI) effective action implements a ladder resummation, which respects thermodynamical consistency and energy conservation [2,3]. These features make the 2PI scheme attractive for nonequilibrium field theory applications [4]. A prerequisite for a nonequilibrium method to be credible is, however, its reliability in equilibrium. There, it is important to check the convergence of expansion series of the 2PI effective action. To this aim the notoriously ill-behaved pressure has been calculated in a scalar context in Ref. [5] showing a monotonous dependence on the coupling constant as well as a relatively small next-to-leading order correction even at couplings of $\mathcal{O}(1)$.

In the framework of gauge theories, however, the implementation of this approximation scheme suffers from various difficulties. One of these is that thermodynamic observables are gauge-fixing independent only up to the order of the truncation. One can illustrate this issue by studying gauge parameter dependence in the Lorentz covariant gauge. For vanishing background fields, the 2PI effective action is a functional of the fermion, gauge and ghost propagators (respectively denoted by D, G and G_{gh}) which also depends on the gauge-fixing parameter ξ: $\Gamma_{\mathrm{2PI}}[D, G, G_{\mathrm{gh}}; \xi]$. The thermal pressure of the system is obtained by evaluating Γ_{2PI} at its stationary point[1] $D = \bar{D}$, $G = \bar{G}$, $G_{\mathrm{gh}} = \bar{G}_{\mathrm{gh}}$, for a given temperature T, and by subtracting the same calculation at zero temperature:

$$\mathcal{P} = -\frac{T}{V}\Gamma_{\mathrm{2PI}}[\bar{D}, \bar{G}, \bar{G}_{\mathrm{gh}}; \xi]\big|_{T=0}^{T}. \tag{1}$$

It is then possible to show that the ξ-dependence of $\mathcal{P}$ uniquely comes from the explicit ξ-dependence of Γ_{2PI} and that it disappears if, in Fourier space,

$$\sum_{\mu\nu} q_\mu q_\nu \bar{G}_{\mu\nu}(q) = \xi. \tag{2}$$

This last equation is the BRST identity for the propagator of the exact theory [6], which may break in a truncated resummation. Indeed, within a given truncation of the 2PI effective action, the BRST symmetry usually does not impose the con-

* Corresponding author.
E-mail addresses: s.borsanyi@sussex.ac.uk (Sz. Borsányi), reinosa@thphys.uni-heidelberg.de (U. Reinosa).

[1] The barred propagators denote the solution of the stationarity equations: $\delta\Gamma_{\mathrm{2PI}}/\delta D = 0$, $\delta\Gamma_{\mathrm{2PI}}/\delta G = 0$ and $\delta\Gamma_{\mathrm{2PI}}/\delta G_{\mathrm{gh}} = 0$.

0370-2693/$ – see front matter
doi:10.1016/j.physletb.2008.02.011

straint (2) above the order of the truncation,[2] leading therefore to ξ-dependent contributions to the pressure.[3]

A certain number of strategies can be put forward in order to try to cope with these inconvenient features. The first possibility is to introduce further approximations, on top of the loop expansion. This is the case of the *approximately self-consistent resummations* introduced in Ref. [10]. Using this method, a gauge-independent determination of the entropy of QCD has been possible and shows a good agreement with lattice results down to temperatures about 2.5 times the transition temperature. There is however no general understanding on how to systematize this approach and evaluate higher orders in a gauge-independent manner.

Another possibility is to stick to the loop expansion of the 2PI effective action but play with the freedom in the choice of field representations. Indeed, the exact theory is invariant under reparametrization of the fields and one could exploit this feature in order to define a loop expansion obeying certain properties. This idea has been discussed in Ref. [11] where it has been applied to the linear sigma model in order to define a systematic loop expansion of the 2PI effective action fulfilling Goldstone's theorem at any order of approximation. Unfortunately no field representation is yet known in gauge theories which would ensure that the BRST identity (2) is fulfilled.

It is finally possible to isolate gauge-independent terms in the expression of the pressure by separating contributions from different perturbative orders. This means a re-expansion of the propagators $\bar{D}$ and $\bar{G}$ in powers of the coupling. The resulting modified resummation scheme did not show a substantial improvement of convergence [12].

A different point of view is based on the experience that the 2PI loop expansion is known to have good convergence properties [5,13]. One can thus expect that contributions above the order of accuracy, and in particular gauge dependences are under control, at least in a large range of coupling values. In this Letter we explore this possibility in QED and compute the pressure (1) from the two-loop truncation of the 2PI effective action using the standard parametrization of the fields. We work in the covariant gauge with arbitrary gauge-fixing parameter ξ, which allows us to study how large gauge-dependent contributions can be.

Before embarking on a numerical evaluation, one has however to pay special attention to a second aspect, namely that of renormalization. The difficulty is related to the fact that truncations of the 2PI effective action only resum particular subclasses of perturbative diagrams for which (perturbative) theorems do not apply. Recently a large effort has been put into extending renormalization theorems to the particular classes of diagrams resummed by the loop expansion of the 2PI effective action. This has been first achieved in the framework of scalar theories [14] as well as scalar theories coupled to a fermionic field [15], and more recently in the framework of QED [7] in the covariant gauge. In this latter case, it is important to emphasize that the renormalization procedure differs substantially from the one in perturbation theory. The reason for this is that, for a given loop truncation of the 2PI effective action and in contrast to what happens in perturbation theory, the photon two- and four-point functions develop longitudinal quantum and thermal corrections. Although these contributions are formally of higher order than the order of the truncation, they bring UV divergences which need to be removed before defining a continuum limit. In Ref. [7] a renormalization procedure involving a new class of counterterms allowed by the gauge symmetry of the theory has been put forward which allows one to deal with this new kind of UV divergences and thus opens the way to practical calculations. In this Letter, we apply these ideas in order to evaluate the pressure of QED from the two-loop 2PI effective action.

Because our purpose is to discuss gauge parameter dependence, it is essential that the considered discretization respects gauge symmetry. In this way, the only source for gauge dependences is the particular truncation we use. For numerical purposes it is also convenient to use lattice rather than dimensional regularization. We thus consider QED on a hypercubic lattice of spacing a. We denote by N_β the number of points on the time direction and N the number of points on each of the spatial directions. The inverse temperature is $\beta = N_\beta a$ and the spatial volume $V = N^3 a^3$. We decompose the lattice action in three pieces: $S = S_g + S_{gf} + S_f$. As gauge-field action, we consider the non-compact action

$$S_g = \frac{1}{4} a^4 \sum_x \sum_{\mu\nu} F_{\mu\nu}(x) F_{\mu\nu}(x), \tag{3}$$

where the field-strength tensor $F_{\mu\nu}(x) = \Delta_\mu^f A_\nu(x) - \Delta_\nu^f A_\mu(x)$ is expressed in terms of the forward derivative[4] $\Delta_\mu^f A_\nu(x) = a^{-1}[A_\nu(x+\hat{\mu}) - A_\nu(x)]$. We use a discretized covariant gauge-fixing term

$$S_{gf} = \frac{1}{2\xi} a^4 \sum_x \sum_{\mu\nu} \Delta_\mu^b A_\mu(x) \Delta_\nu^b A_\nu(x), \tag{4}$$

given in terms of the backward derivative $\Delta_\mu^b A_\nu(x) = a^{-1} \times [A_\nu(x) - A_\nu(x-\hat{\mu})]$ for latter convenience. Finally, the fermionic action is taken to be the naive chiral action

$$S_f = -\frac{1}{2a} a^4 \sum_x \big[\bar{\psi}(x+\hat{\mu})\gamma_\mu U_\mu(x)\psi(x) - \bar{\psi}(x)\gamma_\mu U_\mu^+(x)\psi(x+\hat{\mu})\big], \tag{5}$$

where $U_\mu(x) = \exp(iaeA_\mu(x))$ represents a link variable.

Normally, the interacting two-point function $\bar{D}(x,y)$ or $\bar{G}(x,y)$ corresponds to the correlator of two operators at x and y. On the lattice however, where the fundamental objects

[2] An analysis of similar issues has recently been done in QED [7,8] where it has been shown in particular that, although the 2PI effective action obeys (2PI) Ward identities, these do not impose any constraint on the photon propagator $\bar{G}$. In particular the corresponding polarization tensor is not constrained to be transverse.

[3] More precisely, if one truncates the 2PI effective action at L-loop order, one expects gauge dependences to appear at order e^{2L} [9].

[4] The notation $\hat{\mu}$ stands for the vector of length a along the positive μ direction.