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A note on the viability of Gauss–Bonnet cosmology

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Abstract

In this Letter, we analyze the viability of a vacuum Gauss–Bonnet cosmology by examining the dynamics of the homogeneous and anisotropic background in $4+1$ dimensions. The trajectories of the system either originate from the standard singularity or from non-standard type, the later is characterized by the divergence of time derivative of the Hubble parameters for its finite value. At the onset, the system should relax to Einstein phase at late times as the effect of Gauss–Bonnet term becomes negligible in the low energy regime. However, we find that most of the trajectories emerging from the standard big-bang singularity lead to future re-collapse whereas the system beginning its evolution from the non-standard singularity enters the Kasner regime at late times. This leads to the conclusion that the measure of trajectories giving rise to a smooth evolution from a standard singularity to the Einstein phase is negligibly small for generic initial conditions.

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1. Introduction

Modified theories of gravity are under active consideration at present in cosmology. Efforts are being made to mimic late time acceleration from large scale modification of gravity without resorting to exotic forms of matter dubbed *dark energy* [1,2]. The extra-dimensional effects can give rise to modification of gravity; similar effects can be induced by adding a generic function of Ricci scalar to Einstein–Hilbert action giving rise to $f(R)$ gravity (see Ref. [3] and references therein). The quantum effects can also lead to higher order curvature corrections to Einstein–Hilbert action. These corrections can be systematically computed in perturbative regime of string theory. Amongst all the higher derivative corrections which might arise quantum mechanically, the Gauss–Bonnet (GB) correction has distinguished features [4]. In this case, the equations of motion continue to be of second order thereby ensuring the uniqueness of their solutions. However, in $3+1$ dimensions, the GB term is topological in nature; it acquires dynamics only in higher dimensions. Nevertheless, it can influence the 4-dimensional physics if it is coupled to a dynamically evolving scalar field(s). The pure GB term being in the higher dimensional bulk can also lead to modification of Einstein equations on the brane [5].

Attempts have recently been made to derive current acceleration using the GB term coupled to a scalar field [6–16]. The model exhibits remarkable property that it does not disturb the scaling regime and can give rise to late time transition from matter regime to late time acceleration [7,17]. This beautiful result comes with a cost: the coupling of GB curvature invariant to scalar field gets large at late times and cannot be justified within the perturbative regime the curvature corrections are obtained; the model is also under pressure from nucleosynthesis constraint [7,17]. On the theoretical ground, these models are faced with other serious problems related to stability against perturbations about FRW background [13]. Similar situation is expected to persist in the case of higher order Euler densities coupled to scalar (dilaton/modulus) fields. Of course, one can argue that these fields should be stabilized sufficiently early in order to respect the nucleosynthesis constraints. It is, nevertheless, important to examine the viability of Gauss–Bonnet cosmology in general.

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In this Letter, we take a different route; we consider a vacuum $(4+1)$-dimensional GB cosmology in a homogeneous and anisotropic background and study the structure of generic singularities in the model. Though $4+1$ theories without compactification have no direct applications to our Universe, study of their properties is important for better understanding of gravity in four dimensions, showing its specific properties in comparison with other cases. It is known, for example, that in five-dimensional Einstein gravity the uniqueness theorem for a stationary black hole configurations is no longer valid [18]. Another classical example is related to the disappearance of Mixmaster cosmological chaotic behavior in $10+1$ dimensions [19]. These results have been formulated in the framework of Einstein gravity. The Gauss–Bonnet term can further modify traditional results known for $(3+1)$-dimensional Einstein theory. The main goal of the present Letter is to study the modifications of cosmological singularity due to Gauss–Bonnet term in multidimensional cosmology. In the low energy regime one might expect the system to relax to $(4+1)$-dimensional Kasner geometry. We shall examine the cosmological dynamics of the system under consideration and investigate the measure of trajectories which might connect to Einstein phase at late times.

2. Evolution equations

We consider a $(4+1)$-dimensional theory with the action

$$S=\int \sqrt{-\mathrm{g}}\big(R+\alpha R^2_{\mathrm{GB}}\big)\,d^5x, \tag{1}$$

where R^2_{GB} is the Gauss–Bonnet term

$$R^2_{\mathrm{GB}}=R^{iklm}R_{iklm}-4R^{ik}R_{ik}+R^2.$$

In what follows we shall be interested in the dynamics of the system described by (1) in the homogeneous and anisotropic flat background with the metric

$$\mathrm{g}_{ik}=\mathrm{diag}\big(-n^2(t),a^2(t),b^2(t),c^2(t),d^2(t)\big). \tag{2}$$

This metric provides us a simplest modification of the standard geometry allowing the realization of new dynamical regimes absent in both Einstein gravity and isotropic Gauss–Bonnet modified Einstein theory of gravity (for a complete survey of possible 5-dimensional cosmological backgrounds, see Ref. [20]).

It would be convenient to introduce Hubble parameters with respect to four spatial dimensions $H_{a,b,c,d}=\frac{\dot a,\dot b,\dot c,\dot d}{a,b,c,d}$. In the background described by the metric (2), the action (1) is a functional of the scale factors and the lapse function along with their time derivatives. Varying the action (1) with respect to the lapse function $n(t)$ and setting $n=1$ thereafter we find the constraint equation

$$\begin{aligned}&2H_aH_b+2H_aH_c+2H_aH_d+2H_bH_c+2H_bH_d+2H_cH_d\\&\quad+24\alpha H_aH_bH_cH_d=0,\end{aligned} \tag{3}$$

which is the analogue of Friedmann equation in case of the geometry given by (2). Variation of (1) with respect to scale factors leads to the system of four dynamical equations,

$$\begin{aligned}&2\big(\dot H_b+H_b^2\big)+2\big(\dot H_c+H_c^2\big)+2\big(\dot H_d+H_d^2\big)+2H_bH_c\\&\quad+2H_bH_d+2H_cH_d+8\alpha\big[\big(\dot H_b+H_b^2\big)H_cH_d\\&\quad+\big(\dot H_c+H_c^2\big)H_bH_d+\big(\dot H_d+H_d^2\big)H_bH_c\big]=0,\end{aligned} \tag{4}$$

$$\begin{aligned}&2\big(\dot H_a+H_a^2\big)+2\big(\dot H_c+H_c^2\big)+2\big(\dot H_d+H_d^2\big)+2H_aH_c\\&\quad+2H_aH_d+2H_cH_d+8\alpha\big[\big(\dot H_a+H_a^2\big)H_cH_d\\&\quad+\big(\dot H_c+H_c^2\big)H_aH_d+\big(\dot H_d+H_d^2\big)H_aH_c\big]=0,\end{aligned} \tag{5}$$

$$\begin{aligned}&2\big(\dot H_a+H_a^2\big)+2\big(\dot H_b+H_b^2\big)+2\big(\dot H_d+H_d^2\big)+2H_aH_b\\&\quad+2H_aH_d+2H_bH_d+8\alpha\big[\big(\dot H_a+H_a^2\big)H_bH_d\\&\quad+\big(\dot H_b+H_b^2\big)H_aH_d+\big(\dot H_d+H_d^2\big)H_aH_b\big]=0,\end{aligned} \tag{6}$$

$$\begin{aligned}&2\big(\dot H_a+H_a^2\big)+2\big(\dot H_b+H_b^2\big)+2\big(\dot H_c+H_c^2\big)\\&\quad+2H_aH_b+2H_aH_c+2H_bH_c+8\alpha\big[\big(\dot H_a+H_a^2\big)H_bH_c\\&\quad+\big(\dot H_b+H_b^2\big)H_aH_c+\big(\dot H_c+H_c^2\big)H_aH_b\big]=0.\end{aligned} \tag{7}$$

The evolution equations, in general, look cumbersome for analytical investigations. In what follows we shall investigate the dynamical regimes of the model numerically.

3. Dynamical regimes

The presence of Gauss–Bonnet (GB) term allows some specific dynamical regimes absent in pure Einstein gravity. First of all, the volume of a flat Universe can have local extrema in this background. The another new feature is associated with the possible existence of a non-standard singularity, found in Ref. [21] (this type of singularity was also found previously in another context in Ref. [22], similar situation can also arise in $3+1$-dimensional cosmology with GB-term in presence of a dynamical dilaton [23–25]). Interestingly, the GB brane worlds with the curvature term on the brane can also give rise to this type of singularity [26]. The non-standard singularity, under consideration, is characterized by $\dot H_i\to\infty$ ($H_i=H_{a,b,c,d}$), for finite values of Hubble parameters. It occurs when the major determinant of the system (4)–(7) vanishes.

We note that the generalized Kasner regime, being the solution of vacuum equation motion for Bianchi I Einstein Universe, remains intact in the low-energy regime, when Gauss–Bonnet contribution can be neglected. This solution has the form $ds^2=-dt^2+\sum t^{2p_i}\,dx_i^2$ with two known condition on the power indices

$$\begin{aligned}&p_1^2+p_2^2+p_3^2+p_4^2=1,\\&p_1+p_2+p_3+p_4=1.\end{aligned} \tag{8}$$

In the high-energy regime, the Gauss–Bonnet term becomes important. However, in $4+1$ Universe there are no pure Gauss–Bonnet nontrivial vacuum solutions, similar to found recently for the $(5+1)$-dimensional case [27]. To illustrate this point, let us consider Eq. (3). We observe that there is only one term originating from the Gauss–Bonnet contribution (the last tern on the LHS), so this term and the remaining Einstein contribution (first three terms of the LHS) are equal in absolute values