

Loop diagrams in space with SU(2) fuzziness

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Abstract

The structure of loop corrections is examined in a scalar field theory on a three-dimensional space whose spatial coordinates are noncommutative and satisfy SU(2) Lie algebra. In particular, the 2- and 4-point functions in ϕ^4 scalar theory are calculated at the 1-loop order. The theory is UV-finite as the momentum space is compact. It is shown that the non-planar corrections are proportional to a one-dimensional δ -function, rather than a three-dimensional one, so that in transition rates only the planar corrections contribute.

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1. Introduction

In recent years there has been considerable interest in quantum field theories on noncommutative spaces. This was to a large extent motivated by the observation that this kind of field theories arise in the zero-slope limit of the open string theory in the presence of a constant B-field background [1–4]. In this case the coordinates satisfy the canonical relation

$$[\hat{x}_\mu, \hat{x}_\nu] = i\theta_{\mu\nu}\mathbf{1}, \quad (1)$$

in which θ is an antisymmetric constant tensor and $\mathbf{1}$ represents the unit operator. The theoretical and phenomenological implications of such noncommutative coordinates have been extensively studied, see [5].

One direction to extend studies on noncommutative spaces is to consider spaces where the commutators of the coordinates are not constants. Examples of this kind are the noncommutative cylinder and the q -deformed plane [6], the so-called κ -Poincaré algebra [7–10], and linear noncommutativity of the Lie algebra type [11]. In the latter it is supposed that the dimensionless spatial positions operators satisfy the commutation relations of a Lie algebra [11]:

$$[\hat{x}_a, \hat{x}_b] = f^c_{ab}\hat{x}_c, \quad (2)$$

where f^c_{ab} 's are structure constants of a Lie algebra. One example of this kind is the algebra SO(3), or SU(2). A special case of this is the so-called fuzzy sphere [12,13], where an irreducible representation of the position operators is used which makes the Casimir of the algebra, $(\hat{x}_1)^2 + (\hat{x}_2)^2 + (\hat{x}_3)^2$, a multiple of the identity operator (a constant, hence the name sphere). One can consider the square root of this Casimir as the radius of the fuzzy sphere. This is, however, a noncommutative version of a two-dimensional space (sphere).

In [14,15] a model was introduced in which the representation was not restricted to an irreducible one, instead the whole group was employed. In particular the regular representation of the group, which contains all representations, was considered. As a consequence in such models one is dealing with the whole space, rather than a sub-space, like the case of fuzzy sphere as a 2-dimensional surface. In [14] the basic ingredients for calculus on a linear fuzzy space, as well as the basic notions for

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a field theory on such a space, were introduced. In [15] the basic elements for calculating the matrix elements corresponding to transition between initial and final states were discussed. There the contributions of lowest order (tree level) perturbative expansion of amplitudes were presented for a self-interacting scalar field theory. The models based on the regular representations of $SU(2)$ was treated in more detail, giving the explicit form of the tools and notions introduced in their general forms [14,15].

As mentioned in [14,15], one of the features of models based on linear fuzziness of Lie algebra type is that these theories are free from any ultraviolet divergences if the corresponding Lie group is compact. In fact one can consider the momenta as the coordinates of the group, so that the space of the corresponding momenta is compact iff the group is compact. One important implication of the elimination of the ultraviolet divergences would be that there will be no place for the so-called UV/IR mixing effect [16], which is known to be a common feature of the models based on canonical noncommutativity, the algebra (1).

The purpose of the present work is to examine the structure of the field theory amplitudes at loop order. Here we consider a scalar field theory with ϕ^4 interaction. In particular we consider one-loop corrections to 2- and 4-point functions in this theory. The field theory on a $(2+1)$ -space–time whose coordinates satisfy the Lie algebra of $SO(2,1)$ was studied in [17]. Due to non-compactness of the group in this case, the UV-divergences are present at loop level [17].

The scheme of the rest of this Letter is the following. In Section 2, a brief review is given on basic elements of a field theory on a noncommutative space of $SU(2)$ algebra type. In Sections 3 and 4 the calculation of 2- and 4-point functions are presented, respectively. Section 5 is devoted to concluding remarks; in particular, it is discussed how only the planar sector of the loop corrections contribute to the amplitudes.

2. Field theory on space with $SU(2)$ fuzziness

In [14,15] a model was investigated in a $(3+1)$ -dimensional space–time the dimensionless spatial position operators of which are generators of a *regular* representation of the $SU(2)$ algebra, that is

$$[\hat{x}_a, \hat{x}_b] = \epsilon^c_{ab} \hat{x}_c. \quad (3)$$

As it was discussed in [14], one can use the group algebra as the analogue of functions defined on ordinary space, with group elements $U = \exp(\ell k^a \hat{x}_a)$ as the analogues of $\exp(i\mathbf{k} \cdot \mathbf{x})$, which are a basis for the functions defined on the space. In both cases $\mathbf{k}$ is an ordinary vector with $\mathbf{k} = (k^1, k^2, k^3)$. That is the components of $\mathbf{k}$ are commuting numbers. In the case of noncommutative space, ℓ is a length parameter, and the vector $\mathbf{k}$ is restricted to a ball of radius $(2\pi/\ell)$, with all points of the boundary identified to a single point. The manifold of $\mathbf{k}$ is in fact a 3-sphere. $\mathbf{k}$ can be thought of as the momentum of a particle. The left-right-invariant Haar measure is

$$dU = \frac{\sin^2(\ell k/2)}{(\ell k/2)^2} \frac{d^3 k}{(2\pi)^3}, \quad (4)$$

where $k := |\mathbf{k}|$. The integration region for the coordinates is $k \leq 2\pi/\ell$. We mention that near the origin ($k \ll \ell^{-1}$) the measure is simply $d^3 k / (2\pi)^3$, as it should be. The action of a scalar model with quartic interaction in Fourier space of spatial directions is given by

$$S = \int dt \left\{ \frac{1}{2} \int dU_1 dU_2 [\dot{\phi}(U_1)\dot{\phi}(U_2) + \phi(U_1)O(U_2)\phi(U_2)]\delta(U_1 U_2) - \frac{g}{4!} \int \left[\prod_{j=1}^4 dU_j \right] \phi(U_1)\phi(U_2)\phi(U_3)\phi(U_4)\delta(U_1 U_2 U_3 U_4) \right\}, \quad (5)$$

in which $\dot{\phi}$ is the time derivative of ϕ . In the above,

$$O(U) = c\chi_\lambda(U + U^{-1} - 2\mathbf{1}) - m^2, \quad (6)$$

where c and m are constants, and χ_λ is the character in the representation λ . It is shown that by a proper choice of constant c , near the origin $O(U) \approx -k^2 - m^2$, as it is the case in the ordinary space. The δ -distribution appearing above is simply defined through

$$\int dU \delta(U) f(U) := f(\mathbf{1}), \quad (7)$$

where $\mathbf{1}$ is the identity element of the group. It is easy to see that this delta distribution is invariant under similarity transformations, as well as inversion of the argument:

$$\delta(VUUV^{-1}) = \delta(U), \quad \delta(U^{-1}) = \delta(U). \quad (8)$$

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