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Neutrino flavor instabilities in a time-dependent supernova model

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ABSTRACT

A dense neutrino medium such as that inside a core-collapse supernova can experience collective flavor conversion or oscillations because of the neutral-current weak interaction among the neutrinos. This phenomenon has been studied in a restricted, stationary supernova model which possesses the (spatial) spherical symmetry about the center of the supernova and the (directional) axial symmetry around the radial direction. Recently it has been shown that these spatial and directional symmetries can be broken spontaneously by collective neutrino oscillations. In this letter we analyze the neutrino flavor instabilities in a time-dependent supernova model. Our results show that collective neutrino oscillations start at approximately the same radius in both the stationary and time-dependent supernova models unless there exist very rapid variations in local physical conditions on timescales of a few microseconds or shorter. Our results also suggest that collective neutrino oscillations can vary rapidly with time in the regimes where they do occur which need to be studied in time-dependent supernova models.

1. Introduction

Neutrinos are essential to the thermal, chemical and dynamical evolution of the early universe and some of the compact objects such as the proto-neutron star inside a core-collapse supernova (SN). Whenever there is a difference between the energy spectra and/or fluxes of the electron-flavor neutrino/antineutrino and other neutrino species, the flavor conversion or oscillations among different neutrino flavors can also have important impacts on nucleosynthesis and other physics inside these hot and dense astrophysical environments.

In the absence of collision the flavor evolution of the neutrino obeys the Liouville equation [1–3]

$$\partial_t \rho + \hat{\mathbf{v}} \cdot \nabla \rho = -\mathrm{i}[\mathsf{H}_{\text{vac}} + \mathsf{H}_{\text{mat}} + \mathsf{H}_{\nu\nu},\ \rho], \qquad (1)$$

where $\hat{\mathbf{v}}$ is the velocity of the neutrino, $\rho(t, \mathbf{x}, \mathbf{p})$ is the (Wigner-transformed) flavor density matrices of the neutrino which depends on time t, position $\mathbf{x}$ and neutrino momentum $\mathbf{p}$, H_{vac} is the standard vacuum Hamiltonian, and H_{mat} and $\mathsf{H}_{\nu\nu}$ are the matter and neutrino potentials, respectively. The neutrino potential in Eq. (1) takes the following form [4–6]:

$$\mathsf{H}_{\nu\nu} = \sqrt{2} G_{\mathrm{F}} \int \frac{\mathrm{d}^3 p'}{(2\pi)^3} (1 - \hat{\mathbf{v}} \cdot \hat{\mathbf{v}}')[\rho(t, \mathbf{x}, \mathbf{p}') - \bar{\rho}(t, \mathbf{x}, \mathbf{p}')], \qquad (2)$$

where G_{F} is the Fermi coupling constant, and $\bar{\rho}$ is the density matrix of the antineutrino. Because the neutrino potential couples neutrinos of different momenta, a dense neutrino medium can oscillate in a collective manner (see, e.g., [7–26]; see also [27] for a review).

Eq. (1) poses a challenging 7-dimensional problem (not taking into account the dimensions in neutrino flavors), and it has never been solved in its full form. In previous studies various simplifications have been made so that a numerical or analytic solution to this equation can be found. For neutrino oscillations in SNe a commonly used model is the (neutrino) Bulb model [13]. In this model a spatial spherical symmetry around the center of the SN is imposed so that it has only one spatial dimension. An additional directional axial symmetry around the radial direction is imposed to make the model self-consistent which reduces the number of momentum dimensions to two. One also imposes the time translation symmetry because the timescale of neutrino oscillations is much shorter than those in neutrino emission or dynamic evolution in SNe. However, it has been shown in a series of recent studies that both the spatial and directional symmetries can be broken spontaneously by collective neutrino oscillations if they are not imposed [28–36] (see also [37] for a short review). In both cases small deviations from the initial symmetric conditions are amplified by the symmetry-breaking oscillation modes which can occur closer to the neutrino sphere than the symmetry-preserving modes do.

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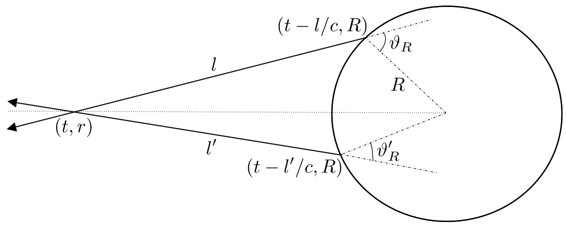

Fig. 1. The geometric picture of the time-dependent (neutrino) Bulb model for supernova. Two neutrinos emitted from the neutrino sphere of radius R with emission angles ϑ_R and ϑ'_R and at time $t - l/c$ and $t - l'/c$ meet each other at radius r and time t, where l and l' are the distances by which the two neutrinos have traveled from the neutrino sphere to their meeting point, respectively.

It is natural to wonder if collective neutrino oscillations can also break the time-translation symmetry spontaneously in SNe [28]. If they do, then a time-independent SN model may not accurately describe the neutrino oscillation phenomenon in SNe even though the typical timescale of the variation in the neutrino emission is much longer than that of neutrino oscillations. In this letter we analyze the neutrino flavor stability in a time-dependent SN model which should provide some interesting insights to this question.

2. Time-dependent neutrino Bulb model

We will focus on the potential differences between the results obtained from the time-dependent and stationary SN models. Therefore, we will employ the time-dependent Bulb model which has the same spatial spherical symmetry and the directional axial symmetry as in the conventional Bulb model. Unlike the conventional stationary Bulb model, however, we will not assume that the emission and flavor evolution of the neutrinos are time-independent (see Fig. 1). For simplicity, we will consider the mixing between two active flavors, the e and x flavors, with the latter being the linear superposition of the μ and τ flavors. We also assume a small vacuum mixing angle $\theta_\mathrm{v} \ll 1$.

It is convenient to use the vacuum oscillation frequency

$$\omega = \pm\frac{|\Delta m^2|}{2E} \tag{3}$$

to label the neutrino and antineutrino with energy E, where Δm^2 is the neutrino mass-squared difference, and the plus and minus signs apply to the neutrino and the antineutrino, respectively. We define reduced neutrino density matrix

$$\varrho(t;r;\omega,u) \propto \begin{cases} \rho & \text{if } \omega > 0, \\ \bar\rho & \text{if } \omega < 0 \end{cases} \tag{4}$$

with normalization

$$\mathrm{tr}\varrho = 1, \tag{5}$$

where $u = \sin^2\vartheta_R$ with ϑ_R being the emission angle of the neutrino on the neutrino sphere (see Fig. 1), and r is the radial distance from the center of the SN.

The equation of motion (EoM) for the (reduced) density matrix ϱ can be written as

$$\mathrm{i}(\partial_t + v_u\partial_r)\varrho = [\mathsf{H}_\mathrm{vac} + \mathsf{H}_\mathrm{mat} + \mathsf{H}_{\nu\nu}, \varrho], \tag{6}$$

where

$$v_u(r) = \sqrt{1 - \left(\frac{R}{r}\right)^2 u} \tag{7}$$

is the radial velocity of the neutrino. In the weak interaction basis the standard vacuum Hamiltonian and the matter potential are

$$\mathsf{H}_\mathrm{vac} \approx -\frac{\eta\omega}{2}\sigma_3 = -\frac{\eta\omega}{2}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \tag{8}$$

and

$$\mathsf{H}_\mathrm{mat} = \lambda\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} \sqrt{2}G_\mathrm{F}n_e & 0 \\ 0 & 0 \end{bmatrix}, \tag{9}$$

respectively, where $\eta = +1$ and -1 for the normal (neutrino mass) hierarchy (NH, i.e. with $\Delta m^2 > 0$) and the inverted hierarchy (IH, $\Delta m^2 < 0$), respectively, and n_e is the net electron number density.

In this letter we assume that the number flux $F_{\nu_\alpha/\bar\nu_\alpha}(E, \vartheta_R)$ of the neutrino/antineutrino in flavor α ($\alpha = e, x$) is time independent [38]. We define the distribution function of the neutrino emission to be

$$g(\omega,u) \propto \left|\frac{\mathrm{d}E}{\mathrm{d}\omega}\right| \times \begin{cases} (F_{\nu_e} + F_{\nu_x}) & \text{if } \omega > 0, \\ -(F_{\bar\nu_e} + F_{\bar\nu_x}) & \text{if } \omega < 0 \end{cases} \tag{10}$$

with normalization conditions

$$\int_0^\infty \mathrm{d}\omega \int_0^1 \frac{\mathrm{d}u}{2}\, g(\omega,u) = 1, \tag{11a}$$

$$\int_{-\infty}^0 \mathrm{d}\omega \int_0^1 \frac{\mathrm{d}u}{2}\, g(\omega,u) = -\frac{N_{\bar\nu}^\mathrm{tot}}{N_\nu^\mathrm{tot}}, \tag{11b}$$

where

$$N_\nu^\mathrm{tot} = \int_0^\infty \mathrm{d}E \int_0^1 \frac{\mathrm{d}u}{2}\,(F_{\nu_e} + F_{\nu_x}), \tag{12a}$$

$$N_{\bar\nu}^\mathrm{tot} = \int_0^\infty \mathrm{d}E \int_0^1 \frac{\mathrm{d}u}{2}\,(F_{\bar\nu_e} + F_{\bar\nu_x}) \tag{12b}$$

are the total number luminosities of the neutrino and antineutrino (i.e. the number of neutrinos or antineutrinos emitted by the whole neutrino sphere per unit time), respectively. The opposite signs of $g(\omega,u)$ for the neutrino and antineutrino in Eq. (10) take into account their different contributions to the neutrino potential in Eq. (1). In the Bulb model the neutrino potential can be written as

$$\mathsf{H}_{\nu\nu}(t;r;u) = \frac{\sqrt{2}G_\mathrm{F}N_\nu^\mathrm{tot}}{4\pi r^2}\int_{-\infty}^\infty \mathrm{d}\omega' \int_0^1 \frac{\mathrm{d}u'}{v_{u'}}(1 - v_u v_{u'}) \times g(\omega',u')\,\varrho(t;r;\omega',u'). \tag{13}$$

Because collective neutrino oscillations usually occur in the regime $R/r \ll 1$ in the Bulb model, we will take the large-radius approximation [39]

$$v_u(r) \approx 1 - \left(\frac{R}{r}\right)^2\frac{u}{2}. \tag{14}$$

In this approximation,

$$\mathsf{H}_{\nu\nu}(t;r;u) \approx \mu\int\left(\frac{u+u'}{2}\right) g'\varrho'\,\mathrm{d}\Gamma', \tag{15}$$

where all the primed quantities are functions of u' and ω', e.g., $\varrho' = \varrho(t;\omega',u';r)$,

$$\mu(r) = \frac{\sqrt{2}G_\mathrm{F}N_\nu^\mathrm{tot}}{4\pi R^2}\left(\frac{R}{r}\right)^4 \tag{16}$$

is the strength of the neutrino potential at radius r, and