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Logarithmic corrected $F(R)$ gravity in the light of Planck 2015

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ABSTRACT

In this Letter, we consider the theory of $F(R)$ gravity with the Lagrangian density $£ = R + \alpha R^2 + \beta R^2 \ln \beta R$. We obtain the constant curvature solutions and find the scalar potential of the gravitational field. We also obtain the mass squared of a scalaron in the Einstein's frame. We find cosmological parameters corresponding to the recent Plank 2015 results. Finally, we analyze the critical points and stability of the new modified theory of gravity and find that logarithmic correction is necessary to have successful model.

1. Introduction

Recent astrophysical observations clarify the accelerated expansion of universe [1–3], which may be described by dark energy scenario. In that case, there are several dark energy models, the simplest one is the cosmological constant, however it is not a dynamical model, so there are another alternative theories such as quintessence [4–9], phantom [10–16], and quintom [17–19] models, or holographic dark energy proposal [20–23]. Moreover, there are interesting models to describe the dark energy such as Chaplygin gas [24–42].

Modification of the Einstein–Hilbert (EH) action through the Ricci scalar can describe inflation and also present accelerated expansion of universe. This called $F(R)$ gravity model, so there are several ways to construct a $F(R)$ gravity models [43–46]. In this paper we consider the particular case of the $F(R)$ gravity model where the Ricci scalar replaced by a new function,

$$F(R) = R + \alpha R^2 + \beta R^2 \ln \beta R, \tag{1.1}$$

where $\beta > 0$ is the parameter with the squared length dimension and also $\alpha > 0$. This model can describe the universe evolution without introducing the dark energy [47], where the cosmic acceleration exists due to the modified gravity. So, $F(R)$ gravity models can be replaced to the cosmological constant model.

The function given by (1.1) without logarithmic correction ($\beta = 0$) has been studied by [48,49] which is applicable to a neutron star with a strong magnetic field [50]. In order to consider effect of gluons in curved space–time, the logarithmic correction in (1.1) proposed by [51]. In the Ref. [51] a phenomenological model based on the equation (1.1) proposed. Motivated by this model, we would like to use relation (1.1) to study some cosmological parameters in the light of new data of Planck 2015.

Initial idea of the $F(R)$ gravity models successfully examined by Refs. [52–54]. Then, several models of $F(R)$ gravity introduced in the literature [55–61]. These are indeed phenomenological models which describe evolution of universe. The Minkowski metric $\eta_{\mu\nu} = \mathrm{diag}(-1, 1, 1, 1)$ and $c = \hbar = 1$ are used in the initial $F(R)$ gravity model [62]. Now, we would like to use logarithmic corrected $F(R)$ model given by the equation (1.1) and exam cosmological consequences of the model using recent data of Planck [63].

This paper is organized as follows. In Section 2, we introduce the model, then we study constant curvature condition in Section 3. In Section 4 we obtain the form of the scalar tensor. Cosmological parameters like tensor to scalar ratio are obtained in Section 5. Critical points and stability analyzed in Section 6. Finally, in Section 7 we give conclusion.

2. The model

We begin with the equation (1.1) to modify the Ricci scalar R in the EH action. The function $F(R)$ satisfies the conditions $F(0) = 0$, corresponding to the flat space–time without cosmological constant. Thus, the action in the Jordan frame becomes

$$S = \int d^4x\sqrt{-g}£ = \int d^4x\sqrt{-g}\left[\frac{1}{2\kappa^2}F(R) + £_m\right], \tag{2.1}$$

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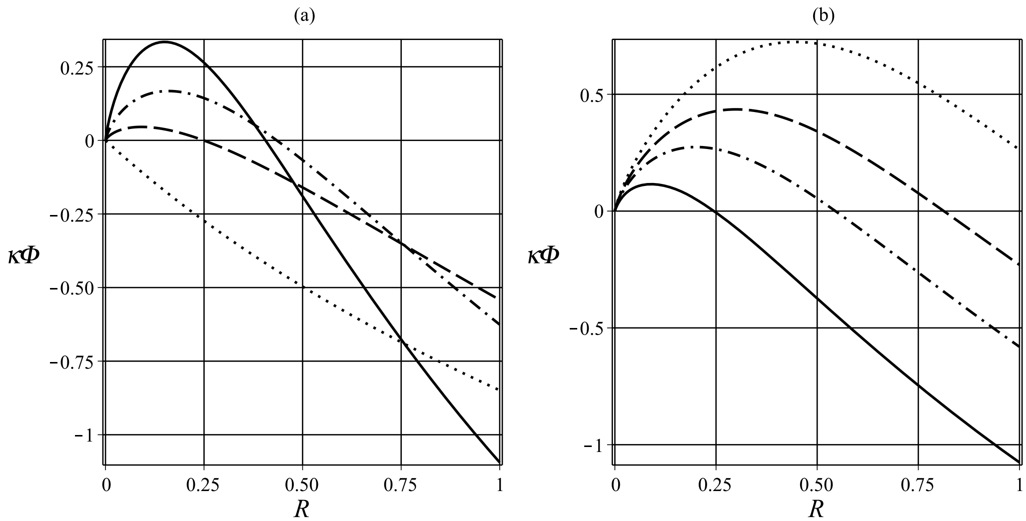

Fig. 1. The function $\kappa\Phi$ versus R. (a) $\alpha = 0.5$, $\beta = 0$ (dot), $\beta = 0.2$ (dash), $\beta = 0.4$ (dot dash), $\beta = 0.8$ (solid). (b) $\beta = 0.5$, $\alpha = 0$ (dot), $\alpha = 0.2$ (dash), $\alpha = 0.4$ (dot dash), $\alpha = 0.8$ (solid).

where $\kappa = M_{pl}^{-1}$, and M_{pl} is the reduced Planck mass, and $\pounds_m$ is the matter Lagrangian density. Our main goal is to study the cosmological parameters describing inflation and the evolution of the early universe. However, we can discuss about consequences in the later time. From the equation (1.1) we obtain

$$F'(R) = 1 + \gamma R + 2\beta R \ln \beta R,$$
$$F''(R) = \lambda + 2\beta \ln \beta R, \tag{2.2}$$

where $\gamma = 2\alpha + \beta$ and $\lambda = 2\alpha + 3\beta$. The function $F(R)$ obeys the quantum stability condition $F''(R) > 0$ for $\alpha > 0$ and $\beta > 0$. This ensures the stability of the solution at high curvature. It follows from the equation (2.2) that the condition of classical stability $F'(R) > 0$ leads to

$$1 + (\gamma + 2\beta \ln \beta R)\, R > 0, \tag{2.3}$$

3. Constant curvature condition

We consider constant curvature solutions of the equations of motion that follow from the action given by (2.1) without matter. The governing equation is given by [64]

$$2F(R) - RF'(R) = 0, \tag{3.1}$$

and hence,

$$R = \frac{1}{\beta}, \tag{3.2}$$

which satisfy $0 < \beta R < 1$. Here, the condition $\frac{F'(R)}{F''(R)} > R$, is satisfied, and therefore the model can describe primordial and present dark energy, which are future stable. From the equation (2.2), we obtain

$$\frac{F'(R)}{F''(R)} = \frac{1 + \gamma R + 2\beta R \ln \beta R}{\lambda + 2\beta \ln \beta R} > R, \tag{3.3}$$

which simplifies to $\beta R < \frac{1}{2}$. Thus, the solution $R_0 = 0$ satisfy the equation (1.1) which then imply that the flat space–time is stable. The second constant curvature solution $\beta R_0 \approx 1$ does not satisfy the equation (3.3), and this leads to unstable de Sitter space–time, so describes inflation.

4. The scalar tensor form

In the Einstein frame corresponding to the scalar tensor theory of gravity, we have the following conformal transformation of the metric [65]:

$$\tilde{g}_{\mu\nu} = F'(R) g_{\mu\nu} = (1 + \gamma R + 2\beta R \ln \beta R) g_{\mu\nu}. \tag{4.1}$$

In that case the action given by the equation (2.1) with $\pounds_m = 0$ written as

$$S = \int d^4x \sqrt{-g}\left[\frac{1}{2\kappa^2}\tilde{R} - \frac{1}{2}\tilde{g}^{\mu\nu}\nabla_\mu \Phi \nabla_\nu \Phi - V(\phi)\right], \tag{4.2}$$

where ∇_μ is the covariant derivative, and $\tilde{R}$ is determined using the conformal metric in the equation (4.1). The scalar field Φ was found to be

$$\Phi = -\sqrt{\frac{3}{2}}\frac{\ln(1 + \gamma R + 2\beta R \ln \beta R)}{\kappa}. \tag{4.3}$$

In Fig. 1 we plotted the function $\kappa\Phi(R)$ for different values of β and α. From Fig. 1 (b) we can see that increasing α decreases value of the scalar field, while there is no regular behavior with variation of β. However the condition $0 < \beta R < 1$ satisfied in the plots. We can see that the scalar field (multiple by κ) is positive for small R and $\beta > 0$.

The potential V was found to be

$$V = \frac{(\gamma - \alpha)R^2 + \beta R^2 \ln \beta R}{2\kappa^2(1 + \gamma R + 2\beta R \ln \beta R)^2}. \tag{4.4}$$

In Fig. 2 we plotted the function $\kappa^2 V$ versus R for different values of the parameters. We can see that there is at least an extremum (minimum) obtained via $V' = 0$ which means

$$\frac{R\,(\lambda + 2\beta \ln(\beta R))}{\kappa^2\,(1 + \gamma R + 2\beta R \ln(\beta R))^2} = 0, \tag{4.5}$$

therefore,

$$2\alpha + 3\beta + 2\beta \ln(\beta R) = 0. \tag{4.6}$$

Thus, using the equation (2.3) and (3.3) in the equation (4.6) with the condition $\beta R < 0.5$, we found that the flat space–time is stable with $R = 0$ and the curvature $R_0 = \frac{1}{\beta} e^{-\frac{3}{2} - \frac{\alpha}{\beta}}$ is unstable.