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Gravitational mass of relativistic matter and antimatter

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ABSTRACT

The universality of free fall, the weak equivalence principle (WEP), is a cornerstone of the general theory of relativity, the most precise theory of gravity confirmed in all experiments up to date. The WEP states the equivalence of the inertial, m, and gravitational, m_g, masses and was tested in numerous occasions with normal matter at relatively low energies. However, there is no confirmation for the matter and antimatter at high energies. For the antimatter the situation is even less clear – current direct observations of trapped antihydrogen suggest the limits $-65 < m_g/m < 110$ not excluding the so-called antigravity phenomenon, i.e. repulsion of the antimatter by Earth. Here we demonstrate an indirect bound $0.96 < m_g/m < 1.04$ on the gravitational mass of relativistic electrons and positrons coming from the absence of the vacuum Cherenkov radiation at the Large Electron–Positron Collider (LEP) and stability of photons at the Tevatron collider in presence of the annual variations of the solar gravitational potential. Our result clearly rules out the speculated antigravity. By considering the absolute potential of the Local Supercluster (LS), we also predict the bounds $1 - 4 \times 10^{-7} < m_g/m < 1 + 2 \times 10^{-7}$ for an electron and positron. Finally, we comment on a possibility of performing complementary tests at the future International Linear Collider (ILC) and Compact Linear Collider (CLIC).

1. Introduction

Since the formulation of the general relativity (GR) by Einstein in 1915–1916 [1,2] there were numerous tests confirming validity of the theory with an exceptional precision [3]. The weak equivalence principle (WEP), postulating the universality of the free fall, or equivalence of the inertial and gravitational masses, was confirmed in torsion balance experiments [4] at the 2×10^{-13} level for the normal matter. The idea of “antigravity” for an exotic matter seems to exist since the end of the XIX century [5], where it appeared together with the idea of antimatter. The modern, quantum, concept of antimatter begins with the theoretical paper of Dirac [6] in 1928 and experimental observation of antielectron (positron) by Anderson [7] in 1933. However, since then, there is no conclusion made about the gravitational interaction of antimatter [8]. The most precise direct observation of cold-trapped antihydrogen [9] sets the limits on the ratio between the inertial m and gravitational m_g masses of the antihydrogen, $-65 < m_g/m < 110$, including systematic errors, at the 5% significance level [9]. At the same time, indirect limits have a long history and are much stricter (even though, most of them use additional assumptions), see review [10] for the arguments prior to 1991. At the moment, the most precise bounds on the difference between the gravitational masses of the matter and antimatter (to our knowledge) are obtained from the comparison of decay parameters of the K^0–$\bar{K}^0$ system [11] (1.8×10^{-9} level with gravitational potential variations and 1.9×10^{-14} with the LS potential) and from comparison of cyclotron frequencies [12] of the p–$\bar{p}$ system [13] (10^{-6} level with LS potential). Equality of the inertial masses for the considered (anti)particles is supported by the $\mathcal{CPT}$-symmetry tested with a much higher precision [14]. These and other indirect limits are, however, not absolute, but relative (between particles and antiparticles) and for relatively low energies. There is, therefore, no guarantee that, e.g., the strange matter (kaons) at any energies, or normal matter and antimatter at high energies (several GeV and higher) will obey WEP. These limits also do not restrict certain WEP violation models, such as the “isotropic parachute model” [15].

Even though astrophysical tests of the Lorentz invariance [16–19] can be, perhaps, used for the precise tests of the WEP (mainly for electrons and protons), they rely on certain models describing the high-energy sources and their dynamics. It is,

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therefore, desirable to obtain similar or better constrains in a well-controlled experimental setup.

In this paper, we constrain possible deviations from WEP for ultrarelativistic electrons and positrons based on the absence of the vacuum Cherenkov radiation from 104.5 GeV electrons and positrons at the LEP at CERN, and on the absence of the photon decays for 340.5 GeV photons at the Tevatron accelerator at Fermilab. It is known that the large Lorentz γ-factor for the ultrarelativistic particles reveals certain gravity and Lorentz-violating effects [20–22], and suppresses the ordinary electromagnetic interaction [23] otherwise overwhelming gravitational forces [24]. This nontrivial fact makes accelerator experiments suitable for the gravitational studies. In addition, continuous collection of the accelerator data makes it possible to study changes in the observables (or exclusion regions in the parameter space) relative to the periodic variations of the astrophysical potentials. This gives one an opportunity to avoid assumptions on the absolute values of the gravitational potentials [11]. An additional advantage of the vacuum Cherenkov radiation for the positron (electron) is its independence of the gravitational properties of the electron (positron). We also choose the electron and positron for our studies because of the absence of an additional internal structure or flavor composition, avoiding possible speculations on, e.g., undiscovered "strange", "isotopic" or "hypercharge" forces [25,26].

2. Dispersion relations

Let us begin with a description of the gravity effects on the high-energy processes. Gravitational field of the Earth (Sun or other distant massive celestial objects) around the accelerator can be considered as homogeneous and described by an isotropic metric for a static weak field,

$$ds^2 = \mathcal{H}^2 dt^2 - \mathcal{H}^{-2}(dx^2 + dy^2 + dz^2)\,, \tag{1}$$

where $\mathcal{H}^2 = 1 + 2\Phi$, and Φ is the gravitational potential, defining the acceleration of free-falling bodies, $\mathbf{a} = -\nabla\Phi(\mathbf{x})$, taken at the Earth's surface.[1] Here and after we work in natural units, $c = \hbar = 1$. We assume that the metric (1) results from a nonrelativistic distribution of normal matter (which is true in the cases considered below), for which the WEP holds with a high precision. Therefore, there is no difference between the inertial and gravitational masses appearing in Eq. (1).

For a massive probe relativistic particle or antiparticle of inertial mass m and gravitational mass m_g (assuming one does not know if they are equal *a priori*), we can write the gravitational potential as

$$\Phi_m = \Phi\,\frac{m_g}{m}\,, \qquad \mathcal{H}_m^2 \equiv 1 + 2\Phi_m\,. \tag{2}$$

This gravitational potential does not appear as a solution to Einstein's equations, but is a way of generalizing the gravitational coupling of the probe massive particles to the background which reproduces the Newton's gravitational law and its relativistic extension [27]. Particles participating in high-energy experiments considered below can be treated as probe particles due to their negligible masses and energies, comparing to the ones of the astrophysical objects creating the background (1). We also do not have a goal of suggesting an alternative action-based theory of gravity, e.g., to take into account the backreaction of the antimatter, since this is not needed with the assumptions made in the paper.

Let us consider a photon with coordinate 4-momentum $\tilde{k}_\mu = (\tilde{\omega}, \tilde{\mathbf{k}})$, and a massive ultrarelativistic particle with coordinate 4-momentum $\tilde{p}_\mu = (\tilde{\mathcal{E}}, \tilde{\mathbf{p}})$ and mass $m \ll \tilde{\mathcal{E}}$. The metric (1) modifies the coordinate speed of light,

$$v_\gamma \equiv |d\mathbf{x}/dt| = \mathcal{H}^2\,, \tag{3}$$

which can be obtained from the null geodesics, $ds^2 = 0$, defining the photon's trajectory. For a massive probe particle moving with the coordinate speed $\tilde{\mathbf{v}}_m$, the line element can be rewritten then as

$$ds^2 = \mathcal{H}_m^2\left(1 - \mathcal{H}_m^{-4}\tilde{\mathbf{v}}_m^2\right)dt^2, \tag{4}$$

and the relativistic action takes the form

$$S = -\int m\,ds = -\int m\mathcal{H}_m\sqrt{1 - \mathcal{H}_m^{-4}\tilde{\mathbf{v}}_m^2}\,dt\,. \tag{5}$$

Using this action, one can easily obtain the coordinate momentum $\tilde{\mathbf{p}}$ and the Hamiltonian (energy) $\tilde{\mathcal{E}}$,

$$\tilde{\mathbf{p}} = \frac{m\mathcal{H}_m^{-3}}{\sqrt{1 - \mathcal{H}_m^{-4}\tilde{\mathbf{v}}_m^2}}\,\tilde{\mathbf{v}}_m\,, \qquad \tilde{\mathcal{E}} = \frac{m\mathcal{H}_m}{\sqrt{1 - \mathcal{H}_m^{-4}\tilde{\mathbf{v}}_m^2}}\,. \tag{6}$$

The modified coordinate dispersion relations for the photon and a massive particle is given then by

$$\tilde{k}^2 = \mathcal{H}^{-4}\tilde{\omega}^2, \quad \tilde{p}^2 = \left(1 + 4|\Phi|\frac{m_g}{m}\right)\left(\tilde{\mathcal{E}}^2 - m^2\right), \tag{7}$$

where $\tilde{k} = |\tilde{\mathbf{k}}|$, $\tilde{p} = |\tilde{\mathbf{p}}|$ and we use $|\Phi|$ instead of $-\Phi$ for the convenience (since potentials of massive bodies are usually taken negative in a coordinate system with the origin in the center of these bodies). The physical expressions can be obtained from the coordinate ones by rescaling, $\mathbf{v} = \mathcal{H}^{-2}\tilde{\mathbf{v}}$, $k = \mathcal{H}\tilde{k}$, $\omega = \mathcal{H}^{-1}\tilde{\omega}$, $p = \mathcal{H}\tilde{p}$, $\mathcal{E} = \mathcal{H}^{-1}\tilde{\mathcal{E}}$, and absorbing the $\mathcal{H}$ factors in (1) into the definitions of the coordinates. We also assume that there is no modification of the physical speed of light within the considered accuracy [3,17,28]. Finally, the physical momenta of the photon and the massive particle take the form

$$k = \omega, \qquad p = \mathcal{E}\left(1 + 2|\Phi|\frac{\Delta m}{m}\right)\sqrt{1 - \frac{m^2}{\mathcal{E}^2}}\,, \tag{8}$$

where $\Delta m = m_g - m$, and we treat $\kappa \equiv 2|\Phi|\Delta m/m$ as a small parameter. Physically, the obtained expressions demonstrate an anomalous redshift the massive particle would get if WEP was violated. This form of the dispersion relations is similar to the ones used in the phenomenology and tests of the quantum gravity and Lorentz violation [22,28–31]. For instance, the dispersion relations (8) can be obtained from the minimal Lorentz-violating Standard Model Extension (SME) [34] with parameters $c_{00} = 3c_{ii} = 3\kappa/4$ (no summation by i) and other parameters set to zero. With the assumption of universality of the speed of light, this is a reasonable approximation as soon as $|\kappa| > 10^{-13}$, which corresponds to the upper boundary on the next dominating SME parameter [47]. Therefore, one can use known tests of the Lorentz-violation (e.g., vacuum Cherenkov radiation, photon decay, synchrotron losses and others) to obtain limits on the parameter κ and, hence, the difference between the gravitational and inertial masses. One of such tests is presented in details in Refs. [29,30] (our κ can be treated as equivalent to their $4c_{00}/3 - \tilde{\kappa}_{tr}$).

[1] The formula for acceleration holds in the nonrelativistic case as well as in the relativistic case if the gravitational forces act perpendicular to the velocity of the particle. If the gravitational force $\mathbf{F} = -\gamma m_g \nabla\Phi$ was parallel to the velocity of the particle $\mathbf{v}_m$, then it would contribute to the acceleration $\mathbf{a}$ with an additional factor $1/\gamma^2$, i.e., $\mathbf{a} = (\mathbf{F} - (\mathbf{v}_m \cdot \mathbf{F})\mathbf{v}_m)/(\gamma m)$, see Ref. [27].