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Spontaneous magnetization of quark matter in the inhomogeneous chiral phase

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ABSTRACT

Considering the density wave of scalar and pseudoscalar condensates, we study the response of quark matter to a weak external magnetic field. In an external magnetic field, the energy spectrum of the lowest Landau level becomes asymmetric about zero, which is closely related to chiral anomaly, and gives rise to the spontaneous magnetization. This mechanism may be one of candidates for the origin of the strong magnetic field in pulsars and/or magnetars.

Recently, the existence of the inhomogeneous chiral phase in the QCD phase diagram has been discussed by the analysis of the effective models such as Nambu–Jona–Lasinio (NJL) model [1–3] or the Schwinger–Dyson approach [4]. In this phase, the quark condensates spatially modulate and it is very similar to the FFLO state in superconductor [5,6] or spin/charge density wave [7,8]. Here, we consider "dual chiral density wave (DCDW)" [1] among many kinds of form of the condensates: the quark condensates then take the form,

$$\Delta(\mathbf{r}) \equiv \langle\bar{\psi}\psi\rangle + i\langle\bar{\psi}i\gamma^5\tau_3\psi\rangle = \Delta e^{iqz}, \tag{1}$$

within the two-flavor QCD. This configuration is also obtained by embedding one of the Hartree–Fock solutions in the NJL_2 model, so-called chiral spiral [9,10]. Since the DCDW phase has been expected to appear in the moderate density region [1], it may be plausible that this phase is realized in neutron stars.

The effect of the magnetic field has been first discussed by Frolov et al. for the DCDW phase [11]. They have found that the spatial direction of the wavevector $\mathbf{q}$ is favored to be parallel to the magnetic field, and the domain of the DCDW phase is much extended in the QCD phase diagram. In Ref. [12] these features arise from some topological effect through spectral asymmetry of the quark energy; quarks exhibit an interesting feature in the presence of the magnetic field and the energy spectrum becomes asymmetric about zero. There also appear new terms in the generalized Ginzburg–Landau expansion due to spectral asymmetry, which signals the novel Lifshitz point in the QCD phase diagram. Thus, they emphasized the peculiar role of the phase degree of freedom of $\Delta(\mathbf{r})$.

Here we further inquire this issue. We study magnetic properties of the DCDW phase to reveal another aspect, *spontaneous magnetization* in the DCDW phase, which suggests a microscopic origin of the strong magnetic field in compact stars.

The origin of the strong magnetic field in compact stars has been one of the long-standing problems. In particular, magnetars have the huge magnetic field $\sim 10^{15}$ G on the surface [13,14]. As a candidate of the origin, amplification of the magnetic field by the dynamo mechanism, magnetorotational instability or the hypothesis of the fossil magnetic field has been proposed so far from the macroscopic point of view. Although numerical simulations have been actively performed, no definite conclusions have been obtained. From the microscopic point of view, it has been proposed that the spontaneous magnetization emerges by spin alignment of quarks on the analogy of the electron gas [15]. However, this phase should be developed in the low density region. As another mechanism, it has been proposed that axial anomaly acting on the parallel layer of the pion domain wall produces magnetization in nuclear matter [16,17].

We use the two-flavor NJL model in the mean field approximation. It is sufficient to consider the each flavor case because La-

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grangian is diagonalized about the flavors. Then, the Dirac Hamiltonian takes the form,

$$H=-i\boldsymbol{\alpha}\cdot\mathbf{D}-2G\gamma^0\left[\frac{1+\gamma_5}{2}\Delta(\mathbf{r})+\frac{1-\gamma_5}{2}\Delta^*(\mathbf{r})\right] \tag{2}$$

with the covariant derivative, $\mathbf{D}=\nabla+ie_f\mathbf{A}$. Taking the external magnetic field $\mathbf{B}$ along the z axis, the energy spectrum constitutes the Landau levels [11],

$$E^f_{p_z n\zeta\epsilon}=\epsilon\sqrt{\left(\zeta\sqrt{p_z^2+m^2}+q/2\right)^2+2|e_fB|n}\quad(n\neq 0), \tag{3}$$

$$E_{p_z\epsilon}=\epsilon\sqrt{p_z^2+m^2}+q/2\quad(n=0), \tag{4}$$

with $\zeta=\pm1$, which denotes the spin polarization. For $m\le q/2$, the sign of ϵ does not always correspond to the particle or anti-particle state because the lower branch of the lowest Landau level (LLL) ($n=0$) is not always negative. In the higher Landau levels (hLLs) ($n\neq0$), there are four energy branches. On the other hand, LLL has only two energy branches and becomes asymmetric about zero. As a result, the thermodynamic potential takes the form in the two-flavor case,

$$\Omega(\mu,T,B;m,q)=\frac{m^2}{4G}+N_c\sum_{f=u,d}\Omega_f, \tag{5}$$

where

$$\Omega_f=-\frac{|e_fB|T}{4\pi}\int\frac{dp_z}{2\pi}\sum_k\left\{\sum_{n,\zeta,\epsilon}\ln\left[\omega_k^2+(E^f_{p_zn\zeta\epsilon}-\mu)^2\right]+\sum_\epsilon\ln\left[\omega_k^2+(E_{p_z\epsilon}-\mu)^2\right]\right\}, \tag{6}$$

with the Matsubara frequency, $\omega_k=(2k+1)\pi T$.

To investigate the response of quark matter to the weak magnetic field B, the thermodynamic potential is expanded about B,

$$\Omega(\mu,T,B;m,q)=\Omega^{(0)}(\mu,T;m,q)+eB\,\Omega^{(1)}(\mu,T;m,q)+\cdots, \tag{7}$$

where e denotes the elementary charge. It should be legitimate as far as $eB/\mu^2, eB/T^2<1$. For μ or T being close to zero, it should be considered that the limit of $B\to0$ is taken before μ or $T\to0$.

Since the vacuum part in $\Omega^{(0)}$ has divergence, it must be regularized by, e.g., the proper time regularization (PTR) [1]. LLL contributes only to $\Omega^{(1)}$ because the energy spectrum does not depend on B and the B dependence only emerges through the Landau degeneracy factor. On the other hand, hLLs contribute to the all order terms of B.

The magnetization can be deduced from the thermodynamic potential as,

$$M=-\frac{\partial\Omega_{\rm min}(\mu,T,B)}{\partial B}, \tag{8}$$

where $\Omega_{\rm min}$ represents the minimized thermodynamic potential about the order parameters and only depends on μ, T and B. Taking the limit, $B\to0$, we find the spontaneous magnetization in the form,

$$M_0=-e\Omega^{(1)}(\mu,T;m=m^{(0)},q=q^{(0)}), \tag{9}$$

where $m^{(0)}$ and $q^{(0)}$ represent the minimal values for $\Omega^{(0)}$. In the following, we will figure out the peculiar role of LLL and show that it leads to spontaneous magnetization.

For the evaluation of $\Omega^{(1)}$, we must carefully treat the effect of chiral anomaly. According to Refs. [18,19], spectral asymmetry generally gives rise to anomalous particle number,

$$N_{\rm anom}=-\lim_{s\to+0}\frac{1}{2}\sum_k{\rm sign}(\lambda_k)|\lambda_k|^{-s}, \tag{10}$$

where λ_k is the eigenvalue of the arbitrary Dirac Hamiltonian. Spectral asymmetry is ill-defined as it is and needs a proper regularization without violating the gauge invariance. In the DCDW phase, LLL exhibits spectral asymmetry to induce anomalous particle number proportional to B [12]. Then, the LLL contribution in $\Omega^{(1)}$ can be decomposed into three terms,

$$\Omega^{(1),{\rm LLL}}=\Omega^{(1),{\rm LLL}}_{\rm vac}+\Omega^{(1),{\rm LLL}}_{\mu}+\Omega^{(1),{\rm LLL}}_{T}, \tag{11}$$

where

$$\Omega^{(1),{\rm LLL}}_{\rm vac}=-\frac{N_c}{4\pi}\int\frac{dp_z}{2\pi}\sum_\epsilon|\omega_\epsilon|, \tag{12}$$

$$\Omega^{(1),{\rm LLL}}_{\mu}=-\frac{N_c}{2\pi}\int\frac{dp_z}{2\pi}\sum_\epsilon(\mu-\omega_\epsilon)\theta(\omega_\epsilon)\theta(\mu-\omega_\epsilon)+\frac{\mu N_c}{4\pi}\eta_H, \tag{13}$$

$$\Omega^{(1),{\rm LLL}}_{T}=-\frac{N_cT}{2\pi}\int\frac{dp_z}{2\pi}\sum_\epsilon\ln\left(1+e^{-\beta|\omega_\epsilon-\mu|}\right), \tag{14}$$

with $\omega_\epsilon=\epsilon\sqrt{p_z^2+m^2}+q/2$. The density dependent term $\Omega^{(1),{\rm LLL}}_{\mu}$ includes the anomalous contribution, $\frac{\mu N_c}{4\pi}\eta_H$, caused by spectral asymmetry. The η-invariant, η_H, renders

$$\eta_H\equiv\lim_{s\to+0}\int\frac{dp_z}{2\pi}\sum_\epsilon|\omega_\epsilon|^{-s}{\rm sign}(\omega_\epsilon)=\begin{cases}-\frac{q}{\pi} & (m>q/2),\\ -\frac{q}{\pi}+\frac{2}{\pi}\sqrt{q^2/4-m^2} & (m<q/2).\end{cases} \tag{15}$$

When $m>q/2$, this quantity agrees with the contribution of the chiral anomaly represented by the Wess–Zumino–Witten (WZW) term [16]. The WZW term does not depend on m but it vanishes in the limit, $m\to0$. The contribution of hLLs to $\Omega^{(1)}$ should be carefully evaluated by expanding the thermodynamic potential with respect to B after the summation over n. Then, the hLLs contribution in $\Omega^{(1)}$ can be similarly decomposed into three terms,

$$\Omega^{(1),{\rm hLL}}=\Omega^{(1),{\rm hLL}}_{\rm vac}+\Omega^{(1),{\rm hLL}}_{\mu}+\Omega^{(1),{\rm hLL}}_{T}, \tag{16}$$

which does not include the anomalous contribution since hLLs have no spectral asymmetry. We find that the three terms are the even function of q, and $\Omega^{(1),{\rm hLL}}_{\rm vac}=-\Omega^{(1),{\rm LLL}}_{\rm vac}$. Thus $\Omega^{(1)}=\Omega^{(1),{\rm LLL}}+\Omega^{(1),{\rm hLL}}$ does not diverge without any regularization and renders

$$\Omega^{(1)}=\frac{\mu N_c}{4\pi}\eta_H-\frac{N_c}{4\pi}\int\frac{dp_z}{2\pi}\sum_\epsilon\sum_{\tau=\pm1}\tau\,(\mu-\tau\omega_\epsilon)\theta(\tau\omega_\epsilon)\theta(\mu-\tau\omega_\epsilon)-\frac{N_cT}{4\pi}\int\frac{dp_z}{2\pi}\sum_\epsilon\sum_{\tau=\pm1}\tau\ln\left(1+e^{-\beta|\omega_\epsilon-\tau\mu|}\right). \tag{17}$$

The first term can be interpreted as the contribution of anomaly and the second and third terms as the contribution of valence quarks. Note that the even function of q in Eq. (11) is completely canceled by the corresponding one in Eq. (16) to make $\Omega^{(1)}$ the odd function of q. It vanishes in the limit: $m\to0$, which behavior