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Moments of ϕ meson spectral functions in vacuum and nuclear matter

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ABSTRACT

Moments of the ϕ meson spectral function in vacuum and in nuclear matter are analyzed, combining a model based on chiral $SU(3)$ effective field theory (with kaonic degrees of freedom) and finite-energy QCD sum rules. For the vacuum we show that the spectral density is strongly constrained by a recent accurate measurement of the $e^+e^- \to K^+K^-$ cross section. In nuclear matter the ϕ spectrum is modified by interactions of the decay kaons with the surrounding nuclear medium, leading to a significant broadening and an asymmetric deformation of the ϕ meson peak. We demonstrate that both in vacuum and nuclear matter, the first two moments of the spectral function are compatible with finite-energy QCD sum rules. A brief discussion of the next-higher spectral moment involving strange four-quark condensates is also presented.

1. Introduction

The study of vector mesons (ρ, ω and ϕ) in nuclear matter has attracted much interest during the last two decades [1,2]. In recent years, the ϕ meson has particularly come into focus, with dedicated experiments investigating its in-medium properties conducted for instance at KEK [3] and at COSY-ANKE [4]. More detailed measurements are being planned for the future in the E16 experiment at J-PARC [5,6]. Interpreting the experimental findings requires a thorough theoretical understanding of the modification of the ϕ meson spectral function at finite density.

One important issue that needs to be understood is whether and how the modifications of the ϕ meson spectral density in nuclear matter reflect changes of the non-perturbative QCD vacuum at finite densities. This question has been investigated previously in the context of QCD sum rules at finite density [7–10]. Using updated input we argue in the present work that the two lowest (zeroth and first) moments are especially suitable for a detailed study of the spectral function with respect to low-dimensional QCD condensates. These lowest moments involve only operators up to dimension 4 which are relatively well understood. Condensates of dimension 6 and higher (such as the four-quark condensates) do not yet enter at that stage. Furthermore, the ratio of the first over the zeroth moment provides a well defined quantity representing a squared mass averaged over the ϕ resonance plus low-energy continuum. This ratio does not depend on details of the spectral function and can in principle be accessed by experimental measurements [11].

This article presents a systematic analysis of the lowest two moments of the ϕ meson spectral function, using finite-energy QCD sum rules (FESR). To describe the spectral function in vacuum we employ a generalized and improved vector dominance model [12,13] and constrain its parameters by recent $e^+e^- \to K^+K^-$ cross section data [14]. The changes of this spectrum in nuclear matter are expressed using updated kaon–nucleon forward scattering amplitudes, with interactions derived from chiral $SU(3)$ effective field theory and coupled channels [15]. The resulting spectral functions are then tested for their consistency with FESR. Also included is a short digression on higher moments and the strange four-quark condensate. A summary and conclusions follow in the final section.

2. Spectral moment analysis in vacuum

2.1. The vacuum spectral function

The starting point is the correlator of the strange quark current, $j_\mu(x) = \frac{1}{3}\bar{s}(x)\gamma_\mu s(x)$, which couples to the physical ϕ meson state:

$$\Pi_{\mu\nu}(q) = i\int d^4x\, e^{iqx}\langle \mathrm{T}[j_\mu(x) j_\nu(0)]\rangle_\rho . \tag{1}$$

$\langle\rangle_\rho$ stands for the expectation value with respect to the ground state of nuclear matter at temperature $T = 0$ and with density ρ. The vacuum case is realized in the limit $\rho = 0$. For a ϕ meson at

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rest it suffices to study the (dimensionless) contracted correlator, $\Pi(q^2) = \frac{1}{3q^2}\Pi^{\mu}_{\mu}(q)$. Using an improved vector dominance model [12], $\mathrm{Im}\,\Pi(q^2)$ can be written as

$$\mathrm{Im}\Pi(q^2) = \frac{\mathrm{Im}\,\Pi_\phi(q^2)}{q^2 g_\phi^2}\left|\frac{(1-a_\phi)q^2 - \mathring{m}_\phi^2}{q^2 - \mathring{m}_\phi^2 - \Pi_\phi(q^2)}\right|^2 . \quad (2)$$

The self-energy $\Pi_\phi(q^2)$ (of dimension *mass*^{2}) is governed by the coupling of the ϕ to $K\overline{K}$ loops and their propagation [12], either in vacuum or in the nuclear medium. The bare mass $\mathring{m}_\phi$ and the coupling constant g_ϕ are determined to agree with experimental observations. The coupling strength is expected to be of the order of the value determined by $SU(3)$ symmetry, $g_\phi \simeq -3g/\sqrt{2}$, with $g = 6.5$. Furthermore, the constant a_ϕ represents the ratio between the $\phi K\overline{K}$ and $\phi\gamma$ couplings and should be close to unity [8,12]. Here we assume $a_\phi = 1$ which gives a very good fit to the experimental $e^+e^- \to K^+K^-$ data as will be shown below. The ϕ self-energy includes the contributions from charged and neutral kaon loops:

$$\Pi_\phi(q^2) = \Pi_{\phi\to K^+K^-}(q^2) + \Pi_{\phi\to K_L^0 K_S^0}(q^2). \quad (3)$$

For specific expressions of the corresponding loop integrals, see [12,13].

The actual values of $\mathring{m}_\phi$ and g_ϕ are determined by fitting Eq. (2) to the recent precise measurement of the $e^+e^- \to K^+K^-$ cross section provided by the BaBar Collaboration [14]. As in this reaction only the charged kaons are detected, only the corresponding $\phi \to K^+K^-$ term of $\mathrm{Im}\,\Pi_\phi(q^2)$ appearing in the numerator of Eq. (2) should be kept while intermediate charge exchange processes, $K^+K^- \leftrightarrow K^0\overline{K}^0$, are included in the resummation of the $K\overline{K}$ loops. In order to describe the data at energies in the continuum above the ϕ meson peak where the simple model of Eq. (2) cannot be expected to work, we add a second order polynomial in $c(q^2) = \sqrt{q^2/q^2_{\mathrm{th}} - 1}$, for $\sqrt{q^2} > \sqrt{q^2_{\mathrm{th}}} = 1040$ MeV:

$$\mathrm{Im}\Pi^{\mathrm{cont.}}(q^2) = A\,c(q^2) + B\,c^2(q^2), \quad (4)$$

with coefficients A and B fitted to the data. This form of the K^+K^- continuum will be kept both in vacuum and nuclear matter. The result of this fit gives $g_\phi = 0.74 \times (-3g/\sqrt{2}) \simeq -10.2$, $\mathring{m}_\phi = 797$ MeV, $A = -5.94 \times 10^{-3}$ and $B = 3.61 \times 10^{-3}$. The respective curve is shown in Fig. 1 together with the experimental data. As demonstrated in this figure, the parameterizations (2), (4) give an accurate description of the data up to about $\sqrt{q^2} = \omega \simeq 1.6$ GeV, above which the experimental points are seen to drop rapidly. This drop is parametrized by a simple linear curve fitted to the data points in this region.

Additional channels beyond $e^+e^- \to K^+K^-$, such as $K^0\overline{K}^0$ and $K\overline{K} + n\pi$ final states, are less well established by empirical data. We include them schematically in the thin solid line shown in Fig. 1.

2.2. Finite-energy sum rules

In the deep-Euclidean limit ($Q^2 = -q^2 \to \infty$) the correlator (1) can be expressed with the help of the operator product expansion (OPE). The following expansion holds in the vacuum:

$$9\,\Pi(q^2 = -Q^2) = -c_0 \log\left(\frac{Q^2}{\mu^2}\right) + \frac{c_2}{Q^2} + \frac{c_4}{Q^4} + \frac{c_6}{Q^6} + \ldots . \quad (5)$$

For the coefficients c_i one finds[1]

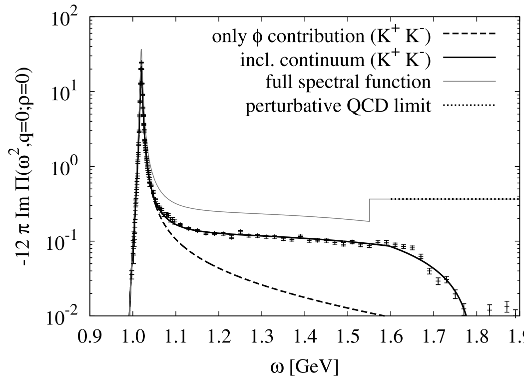

Fig. 1. The fitted spectral function $-12\pi\mathrm{Im}\Pi(\omega^2)$ in vacuum, compared to the experimental data for $\sigma(e^+e^- \to K^+K^-)/\sigma(e^+e^- \to \mu^+\mu^-)$, adapted from [14]. The dashed [solid] curve shows the result when only Eq. (2) [both Eqs. (2) and (4)] are used for the fit. The dotted horizontal line stands for the perturbative QCD limit while the thin gray line represents the full spectral function of Eq. (12), including $K^0\overline{K}^0$ and $K\overline{K} + n\pi$ channels.

$$c_0 = \frac{1}{4\pi^2}\left(1 + \frac{\alpha_s}{\pi}\right), \qquad c_2 = -\frac{3m_s^2}{2\pi^2}, \quad (6)$$

$$c_4 = \frac{1}{12}\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle + 2m_s\langle\bar{s}s\rangle, \quad (7)$$

$$c_6 = -2\pi\alpha_s\Big[\langle(\bar{s}\gamma_\mu\gamma_5\lambda^a s)^2\rangle + \frac{2}{9}\langle(\bar{s}\gamma_\mu\lambda^a s)\sum_{q=u,d,s}(\bar{q}\gamma_\mu\lambda^a q)\rangle\Big] + \frac{m_s^2}{3}\left[\frac{1}{3}\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle - 8m_s\langle\bar{s}s\rangle\right]. \quad (8)$$

Higher order terms in α_s and m_s have also been computed [10]. Here we keep only the most important contributions, sufficient for the purposes of the present work.

Using the once subtracted dispersion relation

$$\Pi(q^2) = \Pi(0) + \frac{q^2}{\pi}\int_0^\infty ds \frac{\mathrm{Im}\Pi(s)}{s(s - q^2 - i\varepsilon)}, \quad (9)$$

and applying the Borel transformation, one derives the sum rule:

$$\frac{1}{M^2}\int_0^\infty ds\,R(s)\,e^{-s/M^2} = c_0 + \frac{c_2}{M^2} + \frac{c_4}{M^4} + \frac{c_6}{2M^6} + \ldots \quad (10)$$

with the spectral function

$$R(s) = -\frac{9}{\pi}\mathrm{Im}\,\Pi(s). \quad (11)$$

At large s this spectral function approaches its perturbative QCD limit, so the following ansatz is introduced:

$$R(s) = R_\phi(s)\,\Theta(s_0 - s) + R_{\mathrm{c}}(s)\,\Theta(s - s_0), \quad (12)$$

with $R_{\mathrm{c}}(s) = c_0$, and s_0 represents a scale that delineates the low-energy and high-energy parts of the spectrum. Substituting this into Eq. (10) and expanding the left-hand side in inverse powers of M^2, one derives the finite-energy sum rules:

$$\int_0^{s_0} ds\,R_\phi(s) = c_0 s_0 + c_2, \quad (13)$$

$$\int_0^{s_0} ds\,s\,R_\phi(s) = \frac{c_0}{2}s_0^2 - c_4, \quad (14)$$

[1] The λ_a in c_6 denote Gell-Mann $SU(3)$ color matrices.