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Correction terms for the thermodynamics of a black Saturn

Mir Faizal [a,*], Behnam Pourhassan [b]

[a] Department of Physics and Astronomy, University of Waterloo, Waterloo, ON N2L 3G1, Canada
[b] School of Physics, Damghan University, Damghan, Iran

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ABSTRACT

In this paper, we will analyze the effects of thermal fluctuations on the stability of a black Saturn. The entropy of the black Saturn will get corrected due to these thermal fluctuations. We will demonstrate that the correction term generated by these thermal fluctuations is a logarithmic term. Then we will use this corrected value of the entropy to obtain bounds for various parameters of the black Saturn. We will also analyze the thermodynamical stability of the black Saturn in presence of thermal fluctuations, using this corrected value of the entropy.

1. Introduction

If entropy is not associated with a black hole, then the entropy of the universe will spontaneous reduce whenever an object with a finite entropy crosses the horizon. Thus, entropy has to be associated with a black hole to prevent the violation of the second law of thermodynamics [1,2]. In fact, black holes have more entropy than any other object of the same volume [3,4]. This prevents the violation of second law of thermodynamics. This maximum entropy of the black holes is proportional to the area of the horizon [5]. Thus, if S is the entropy associated with a black hole, and A is the area of the horizon, then the relation between S and A can be expressed as $S = A/4$. The observation that the entropy scales with the area of the black hole, instead of its volume, has motivated the development of the holographic principle [6,7]. The holographic principle states that the degrees of freedom in a region of space are the same as the degrees of freedom on the boundary surrounding that region of space. The geometry of black holes will undergo quantum fluctuations. These quantum corrections will lead to thermal fluctuations. These thermal fluctuations will in turn generate correction terms for various thermodynamical quantities associated with black holes [8,9]. Thus the holographic principle can get modified near Planck scale [10,11]. It may be noted that even though the thermodynamics of black holes is expected to get corrected due to thermal fluctuations, we can neglect such correction terms for large black holes. This is because these thermal fluctuations occur because of quantum fluctuations of the geometry of space–time, and such quantum fluctuations can be neglected for large black holes. However, as the black holes radiate Hawking radiation, they tend to evaporate in course of time. Then the size of the black holes reduces in course of time due to the Hawking radiation. As the black holes become smaller the quantum fluctuations give more dominating contribution to the geometry of space–time. Thus, the thermal fluctuations cannot be neglected for small black holes, or for black hole at the last stages of their evaporation. The correction terms to the entropy of black holes coming from thermal fluctuations have been calculated. It has been demonstrated that these correction terms are expressed as logarithmic functions of the original thermodynamic quantities.

The corrections to the thermodynamics of black holes have also been calculated using the density of microstates for asymptotically flat black holes [12]. This analysis has been done in the framework of non-perturbative quantum general relativity. Here conformal blocks of a well defined conformal field theory are associated with the density of states for a black hole. This density of states is then used to calculate the relation between the entropy of a black hole and the area of its horizon. The leading order relation between the entropy of a black hole and the area of its horizon is observed to be the standard Bekenstein entropy area relation for the large black holes. However, this relation between the area and entropy of a black hole gets corrected in this analysis. The leading order correction terms to the entropy of the black hole are demonstrated to be logarithmic corrections. It may be noted that such correction terms have also been calculated using the Cardy formula [13]. In fact, it has been demonstrated using this formula that such logarithmic correction terms will be generated for all

* Corresponding author.
E-mail addresses: f2mir@uwaterloo.ca (M. Faizal), b.pourhassan@du.ac.ir (B. Pourhassan).

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black holes whose microscopic degrees of freedom are described by a conformal field theory. The correction terms to the entropy of a BTZ black hole have been calculated using such logarithmic exact partition function [14]. It has been again observed that these correction terms can be expressed using logarithmic functions. It has also been possible to obtain logarithmic correction terms for the entropy of a black hole by analyzing matter fields in backgrounds of a black hole [15–17].

The correction terms generated from string theoretical effects can also be expressed using logarithmic functions [18–21]. The logarithmic correction terms for the entropy of a dilatonic black holes have been calculated [22]. Finally, the expansion of the partition function has also been used to calculate the correction terms for the entropy of a black hole [23]. Such correction terms obtained by using the expansion of the partition function again are logarithmic correction terms. The correction to the thermodynamics of black holes from generalized uncertainty principle has also been studied [24]. In this analysis the thermodynamics of the black holes gets modifies due to the generalization of the usual Heisenberg uncertainty principle. It has been demonstrated this modified thermodynamics of the black holes predicts the existence of a remnant for black holes. The existence of such remnants for black holes can have important phenomenological consequences [25].

As the quantum fluctuations can occur in all black hole geometries, we expect that the thermodynamics of all black objects will get corrected due to thermal fluctuations. Thus, we can use the modified relation between the entropy and area to analyze the corrections for the thermodynamics of any black object. In this paper, we will analyze such correction terms for the thermodynamics of black Saturn. The black Saturns are solutions to Einstein equations in higher dimensions. They are described by a black hole surrounded by a black ring [26,27]. This black ring is in thermodynamical equilibrium with a spherical black hole. The thermodynamics of black Saturn has been studied [28]. The thermodynamic equilibrium is obtained because of the rotation of the black ring. It is also possible to construct a black Saturn with a static black ring [29,30]. In this case, the system remains in thermodynamic equilibrium because of an external magnetic field. It may be noted that conditions for meta-stability of a black Saturn have also been studied [31]. It has been demonstrated that the black Saturn is causal stably on the closure of the domain of outer communications [32]. The relation between the black Saturn and Myers–Perry black hole has also been analyzed [33]. It may be noted that the thermodynamics of a charged dilatonic black Saturn has also been studied [34]. It is expected that both the black hole and black ring in a black Saturn will reduce in size due to the Hawking radiation. Thus, at a certain stage quantum fluctuations in the geometry of a black Saturn will also become important. To analyze the effect of these quantum fluctuations in the geometry of a black Saturn, we will need to analyze the thermal fluctuations in the thermodynamics of black Saturn. So, we will study the corrections to the thermodynamics of a black Saturn by considering thermal fluctuations around the equilibrium.

2. Black Saturn

In this section, we will review the thermodynamics of black Saturn. The metric for black Saturn can be written as [26]

$$ds^2 = -\frac{H_y}{H_x}\left[dt + (\frac{\omega_\psi}{H_y} + q)d\psi\right]^2 + H_x\left[k^2 P(d\rho^2 + dz^2) + \frac{G_y}{H_y}d\psi^2 + \frac{G_x}{H_x}d\varphi^2\right], \quad (1)$$

where q and k are constants, and

$$G_x = \frac{\mu_4}{\mu_3\mu_5}\rho^2$$
$$G_y = \frac{\mu_3\mu_5}{\mu_4}. \quad (2)$$

Here we have used

$$P = (\mu_3\mu_4 + \rho^2)^2(\mu_1\mu_5 + \rho^2)(\mu_4\mu_5 + \rho^2), \quad (3)$$

and

$$\mu_i = \sqrt{\rho^2 + (z - a_i)^2} - (z - a_i) = R_i - (z - a_i). \quad (4)$$

The real constant parameters a_i $(i = 1, \dots, 5)$ satisfy the following condition,

$$a_1 \leq a_5 \leq a_4 \leq a_3 \leq a_2. \quad (5)$$

Furthermore, we also have

$$H_x = \frac{M_0 + c_1^2 M_1 + c_2^2 M_2 + c_1 c_2 M_3 + c_1^2 c_2^2 M_4}{F}$$
$$H_y = \frac{1}{F}\frac{\mu_3}{\mu_4}\left[\frac{\mu_1}{\mu_2}M_0 - c_1^2 M_1 \frac{\rho^2}{\mu_1\mu_2} - c_2^2 M_2 \frac{\mu_1\mu_2}{\rho^2} + c_1 c_2 M_3 + c_1^2 c_2^2 M_4 \frac{\mu_2}{\mu_1}\right], \quad (6)$$

where c_1 and c_2 are real constants, and

$$M_0 = \mu_2\mu_5^2(\mu_1 - \mu_3)^2(\mu_2 - \mu_4)^2(\rho^2 + \mu_1\mu_2)^2 \times (\rho^2 + \mu_1\mu_4)^2(\rho^2 + \mu_2\mu_3)^2,$$
$$M_1 = \mu_1^2\mu_2\mu_3\mu_4\mu_5\rho^2(\mu_1 - \mu_2)^2(\mu_2 - \mu_4)^2 \times (\mu_1 - \mu_5)^2(\rho^2 + \mu_2\mu_3)^2,$$
$$M_2 = \mu_2\mu_3\mu_4\mu_5\rho^2(\mu_1 - \mu_2)^2(\mu_1 - \mu_3)^2 \times (\rho^2 + \mu_1\mu_4)^2(\rho^2 + \mu_2\mu_5)^2,$$
$$M_3 = 2\mu_1\mu_2\mu_3\mu_4\mu_5(\mu_1 - \mu_3)(\mu_1 - \mu_5) \times (\mu_2 - \mu_4)(\rho^2 + \mu_1^2)(\rho^2 + \mu_2^2) \times (\rho^2 + \mu_1\mu_4)(\rho^2 + \mu_2\mu_3)(\rho^2 + \mu_2\mu_5),$$
$$M_4 = \mu_1^2\mu_2\mu_3^2\mu_4^2(\mu_1 - \mu_5)^2(\rho^2 + \mu_1\mu_2)^2(\rho^2 + \mu_2\mu_5)^2, \quad (7)$$

with

$$F = \mu_1\mu_5(\mu_1 - \mu_3)^2(\mu_2 - \mu_4)^2(\rho^2 + \mu_1\mu_3) \times (\rho^2 + \mu_2\mu_3)(\rho^2 + \mu_1\mu_4)(\rho^2 + \mu_2\mu_4)(\rho^2 + \mu_2\mu_5) \times (\rho^2 + \mu_3\mu_5)(\rho^2 + \mu_1^2)(\rho^2 + \mu_2^2)(\rho^2 + \mu_3^2) \times (\rho^2 + \mu_4^2)(\rho^2 + \mu_5^2). \quad (8)$$

Here ω_ψ is expressed as

$$\omega_\psi = \frac{2}{F\sqrt{G_x}}\left[c_1 R_1\sqrt{M_0 M_1} - c_2 R_2\sqrt{M_0 M_2} + c_1^2 c_2 R_2\sqrt{M_1 M_4} - c_1 c_2^2 R_1\sqrt{M_2 M_4}\right], \quad (9)$$

where R_1 and R_2 are given in the relation (4). Free parameters of the model are fixed by [27] as

$$L^2 = a_2 - a_1, \quad (10)$$

and

$$c_1 = \pm\sqrt{\frac{2(a_3 - a_1)(a_4 - a_1)}{a_5 - a_1}}. \quad (11)$$