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Relativistic second-order dissipative hydrodynamics at finite chemical potential

Amaresh Jaiswal [a,*], Bengt Friman [a], Krzysztof Redlich [b,c]

[a] GSI, Helmholtzzentrum für Schwerionenforschung, Planckstrasse 1, D-64291 Darmstadt, Germany
[b] Institute of Theoretical Physics, University of Wroclaw, PL-50204 Wroclaw, Poland
[c] Extreme Matter Institute EMMI, GSI, Planckstrasse 1, D-64291 Darmstadt, Germany

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ABSTRACT

Starting from the Boltzmann equation in the relaxation time approximation and employing a Chapman–Enskog like expansion for the distribution function close to equilibrium, we derive second-order evolution equations for the shear stress tensor and the dissipative charge current for a system of massless quarks and gluons. The transport coefficients are obtained exactly using quantum statistics for the phase space distribution functions at non-zero chemical potential. We show that, within the relaxation time approximation, the second-order evolution equations for the shear stress tensor and the dissipative charge current can be decoupled. We find that, for large values of the ratio of chemical potential to temperature, the charge conductivity is small compared to the coefficient of shear viscosity. Moreover, we show that in the relaxation-time approximation, the limiting behaviour of the ratio of heat conductivity to shear viscosity is qualitatively similar to that obtained for a strongly coupled conformal plasma.

1. Introduction

High-energy heavy ion collisions at the BNL Relativistic Heavy Ion Collider (RHIC) [1,2] and the CERN Large Hadron Collider (LHC) [3–5] create strongly interacting matter under extreme conditions of high temperature and density as it is believed to have existed in the very early universe [6,7]. At such conditions, quarks and gluons are deconfined to form a new state of matter, the quark–gluon plasma (QGP). The QGP behaves as a strongly coupled plasma having the smallest shear viscosity-to-entropy density ratio, η/s [8–13]. Relativistic hydrodynamics has been applied quite successfully to describe the space–time evolution of the QGP formed in high-energy heavy ion collisions and to estimate its transport coefficients [14].

In applications of hydrodynamics it is rather straightforward to employ the ideal (Euler) equations. The inclusion of dissipative effects in the evolution of the QGP started only a few years ago. However, most of the studies have focused on exploring the effects of the shear viscosity on the QGP evolution and extracting its magnitude from experimental measurements. Nevertheless, there are other sources of dissipation such as bulk viscous pressure and dissipative charge current that may have a significant effect on the hydrodynamic evolution of the QGP. While the effects of bulk viscous pressure has been studied in some details [15–20], the dissipative charge current has been largely ignored. This may be attributed to the fact that at very high energies, baryon number and its corresponding chemical potential are negligible. However, at lower collision energies such as those probed in the RHIC low-energy scan or at the upcoming experiments at the Facility for Antiproton and Ion Research (FAIR), baryon number can no longer be ignored and therefore charge diffusion may play an important role.

The earliest theoretical formulations of relativistic dissipative hydrodynamics are due to Eckart [21] and Landau–Lifshitz [22]. However these formulations, collectively called relativistic Navier–Stokes theory, involve only first-order gradients and suffer from acausality and numerical instability due to the parabolic nature of the equations. Second order or extended theories by Grad [23], Müller [24] and Israel and Stewart (IS) [25] were introduced to restore causality. Therefore it is imperative that second order dissipative hydrodynamic equations should be employed in order to correctly describe the evolution of the QGP. However, the IS formulation of a causal theory of relativistic hydrodynamics from kinetic theory, contains several inconsistencies and approximations, the resolution of which is currently an active research area [26–38].

* Corresponding author.
E-mail address: jaiswal.amaresh@gmail.com (A. Jaiswal).

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In order to formulate a causal theory of relativistic dissipative hydrodynamics from kinetic theory, it is desirable to first specify the form of the non-equilibrium phase–space distribution function. For a system close to local thermodynamic equilibrium, the non-equilibrium corrections to the distribution function can be obtained using either (i) Grad's moment method [23] or (ii) the Chapman–Enskog method [39]. Although both methods involve expanding the distribution function around its equilibrium value, it has been demonstrated that the Chapman–Enskog method in the relaxation time approximation results in a better agreement with microscopic Boltzmann simulations [32,33] as well as with exact solutions of the Boltzmann equation in the relaxation-time approximation [32–36].

In the absence of conserved charges, the Chapman–Enskog method has been used to compute the second-order transport coefficients for vanishing [32–34] as well as finite particle masses [35,36]. On the other hand, in the presence of conserved charges but for classical particles with vanishing masses, the second-order transport coefficients corresponding to charge diffusion (or alternatively heat conduction) have been obtained by employing the moment method [40,41]. However, they still remain to be determined for quantum statistics. Here, we employ the Chapman–Enskog method to achieve this.

In this Letter, we present the derivation of second-order evolution equations for shear stress tensor and dissipative charge current for a system consisting of massless quarks and gluons. In order to obtain the form of the non-equilibrium distribution function, we employ a Chapman–Enskog like expansion to iteratively solve the Boltzmann equation in the relaxation time approximation [32]. Using this expansion, we derive the first-order constitutive relations and subsequently the second-order evolution equations for the dissipative quantities. The transport coefficients are obtained exactly using quantum statistics for the quark and gluon phase–space distribution functions with a non-vanishing quark chemical potential. Moreover, we show that, up to second-order in the gradient expansion, the evolution equations for the shear stress tensor and the dissipative charge current can be decoupled. We also find that, for large values of the ratio of chemical potential to temperature, the charge conductivity is small compared to the coefficient of shear viscosity. Finally we demonstrate that the limiting behaviour of the heat conductivity to shear viscosity ratio, obtained here in the relaxation-time approximation, is qualitatively identical to that of a conformal fluid in the strong coupling limit.

2. Relativistic hydrodynamics

In the case of massless partons, i.e., massless quarks and gluons, the conserved energy–momentum tensor and the net-quark current can be expressed in terms of the single particle phase–space distribution function as [42]

$$T^{\mu\nu} = \int dp\, p^\mu p^\nu \left[g_q(f_q + f_{\bar q}) + g_g f_g\right] = \epsilon u^\mu u^\nu - P\Delta^{\mu\nu} + \pi^{\mu\nu}, \tag{1}$$

$$N^\mu = \int dp\, p^\mu \left[g_q(f_q - f_{\bar q})\right] = n u^\mu + n^\mu, \tag{2}$$

where $dp = d\mathbf{p}/[(2\pi)^3|\mathbf{p}|]$, p^μ is the particle four momenta, and g_q and g_g are the quark and gluon degeneracy factor, respectively. Here f_q, $f_{\bar q}$, and f_g are the phase–space distribution functions for quarks, anti-quarks, and gluons. In the tensor decompositions, ϵ, P, and n are the energy density, pressure, and the net quark number density. The projection operator $\Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu$ is orthogonal to the hydrodynamic four-velocity u^μ defined in the Landau frame: $T^{\mu\nu}u_\nu = \epsilon u^\mu$. We work with the Minkowskian metric tensor $g^{\mu\nu} \equiv \mathrm{diag}(+,-,-,-)$.

The dissipative quantities in Eqs. (1) and (2) are the shear stress tensor $\pi^{\mu\nu}$ and the particle diffusion current n^μ. With the definition of the energy–momentum tensor in Eq. (1), the bulk viscous pressure vanishes in the massless case. The energy–momentum conservation, $\partial_\mu T^{\mu\nu} = 0$, and particle four-current conservation, $\partial_\mu N^\mu = 0$, yields the fundamental evolution equations for ϵ, u^μ and n, as

$$\dot\epsilon + (\epsilon + P)\theta - \pi^{\mu\nu}\sigma_{\mu\nu} = 0, \tag{3}$$

$$(\epsilon + P)\dot u^\alpha - \nabla^\alpha P + \Delta^\alpha_\nu \partial_\mu \pi^{\mu\nu} = 0, \tag{4}$$

$$\dot n + n\theta + \partial_\mu n^\mu = 0. \tag{5}$$

Here we use the standard notation $\dot A = u^\mu\partial_\mu A$ for co-moving derivatives, $\theta \equiv \partial_\mu u^\mu$ for the expansion scalar, $\sigma^{\mu\nu} \equiv \frac{1}{2}(\nabla^\mu u^\nu + \nabla^\nu u^\mu) - \frac{1}{3}\theta\Delta^{\mu\nu}$ for the velocity stress tensor, and $\nabla^\alpha = \Delta^{\mu\alpha}\partial_\mu$ for space-like derivatives.

In the following, we briefly outline the thermodynamic properties of a QGP in equilibrium. In this case, the phase–space distribution functions for quarks, anti-quarks and gluons are given by

$$f_q^{(0)} = \frac{1}{\exp(\beta\, u\cdot p - \alpha) + 1}, \tag{6}$$

$$f_{\bar q}^{(0)} = \frac{1}{\exp(\beta\, u\cdot p + \alpha) + 1}, \tag{7}$$

$$f_g^{(0)} = \frac{1}{\exp(\beta\, u\cdot p) - 1}, \tag{8}$$

respectively, where $u\cdot p \equiv u_\mu p^\mu$, $\beta = 1/T$ is the inverse temperature and $\alpha = \mu/T$ is the ratio of the quark chemical potential to temperature. We consider vanishing chemical potential for gluons because they are unconstrained by the conservation laws.

The temperature, T, and chemical potential, μ, of the system are determined by the matching condition $\epsilon = \epsilon_0$ and $n = n_0$, where ϵ_0 and n_0 are the energy density and the net quark number density in equilibrium. The energy density, pressure and the net quark number density for a system of massless quarks and gluons in equilibrium are given by

$$\epsilon_0 \equiv u_\mu u_\nu \int dp\, p^\mu p^\nu \left[g_q\left(f_q^{(0)} + f_{\bar q}^{(0)}\right) + g_g f_g^{(0)}\right] = \frac{(4g_g + 7g_q)\pi^2}{120}T^4 + \frac{g_q}{4}T^2\mu^2 + \frac{g_q}{8\pi^2}\mu^4 \tag{9}$$

$$P_0 \equiv -\frac{1}{3}\Delta_{\mu\nu} \int dp\, p^\mu p^\nu \left[g_q\left(f_q^{(0)} + f_{\bar q}^{(0)}\right) + g_g f_g^{(0)}\right] = \frac{(4g_g + 7g_q)\pi^2}{360}T^4 + \frac{g_q}{12}T^2\mu^2 + \frac{g_q}{24\pi^2}\mu^4 \tag{10}$$

$$n_0 \equiv u_\mu \int dp\, p^\mu \left[g_q\left(f_q^{(0)} - f_{\bar q}^{(0)}\right)\right] = \frac{g_q}{6}T^2\mu + \frac{g_q}{6\pi^2}\mu^3. \tag{11}$$

The equilibrium entropy density then becomes

$$s_0 \equiv \frac{\epsilon_0 + P_0 - \mu n_0}{T} = \frac{(4g_g + 7g_q)\pi^2}{90}T^3 + \frac{g_q}{6}T\mu^2. \tag{12}$$

The above expressions for ϵ_0, P_0, n_0, and s_0 can also be obtained directly from the partition function of an ideal QGP [42],

$$\ln Z = \frac{V}{T}\left[\frac{(4g_g + 7g_q)\pi^2}{360}T^4 + \frac{g_q}{12}T^2\mu^2 + \frac{g_q}{24\pi^2}\mu^4\right], \tag{13}$$