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Flavor instabilities in the neutrino line model

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ABSTRACT

A dense neutrino medium can experience collective flavor oscillations through nonlinear neutrino–neutrino refraction. To make this multi-dimensional flavor transport problem more tractable, all existing studies have assumed certain symmetries (e.g., the spatial homogeneity and directional isotropy in the early universe) to reduce the dimensionality of the problem. In this work we show that, if both the directional and spatial symmetries are not enforced in the neutrino line model, collective oscillations can develop in the physical regimes where the symmetry-preserving oscillation modes are stable. Our results suggest that collective neutrino oscillations in real astrophysical environments (such as core-collapse supernovae and black-hole accretion discs) can be qualitatively different from the predictions based on existing models in which spatial and directional symmetries are artificially imposed.

1. Introduction

Neutrinos are influential in many hot and dense astrophysical environments where they are copiously produced. For example, 99% of the immense power of a core-collapse supernova (SN) is carried away by $\sim 10^{58}$ neutrinos within just ~ 10 seconds (see, e.g., [1] for a review). Through the reactions

$$\nu_e + n \rightleftharpoons p + e^-, \qquad \bar{\nu}_e + p \rightleftharpoons n + e^+ \tag{1}$$

electron-flavor neutrinos extract energy from and deposit energy into the environment and change the n-to-p ratio of the baryonic matter.

It has been firmly established by various experiments that neutrinos can oscillate among different flavors or weak interaction states during propagation. Most of the neutrino mixing parameters have been determined, although it is still unknown whether the neutrino has a normal mass hierarchy (NH) or an inverted one (IH), i.e. whether $|\nu_3\rangle$ is the most massive of the three neutrino mass eigenstates $|\nu_i\rangle$ $(i = 1, 2, 3)$ or not (see, e.g., [2] for a review). The Mikheyev–Smirnov–Wolfenstein (MSW) [3,4] flavor transformation of neutrinos in ordinary matter is also well understood. However, because of its nonlinearity, our understanding of neutrino oscillations in dense neutrino media (such as the one surrounding the proto-neutron star in SN) is still elementary and requires more investigation.

In the absence of collisions the flavor evolution of neutrinos is described by the Liouville equations [5–7]

$$\partial_t \rho + \hat{\mathbf{v}} \cdot \nabla \rho = -\mathrm{i}[\mathsf{H}_0 + \mathsf{H}_{\nu\nu},\ \rho], \tag{2a}$$

$$\partial_t \bar{\rho} + \hat{\mathbf{v}} \cdot \nabla \bar{\rho} = -\mathrm{i}[\bar{\mathsf{H}}_0 + \mathsf{H}_{\nu\nu},\ \bar{\rho}], \tag{2b}$$

where $\hat{\mathbf{v}}$ is the velocity of the neutrino, $\rho(t, \mathbf{x}, \mathbf{p})$ is the (Wigner-transformed) neutrino flavor density matrix which is a function of time t, position $\mathbf{x}$ and neutrino momentum $\mathbf{p}$, H_0 is the Hamiltonian in the absence of ambient neutrinos, and $\mathsf{H}_{\nu\nu}$ is the neutrino(-neutrino coupling) potential. Throughout this letter the physical quantities with bars such as $\bar{\rho}$ and $\bar{\mathsf{H}}_0$ are for antineutrinos. We assume that neutrinos are relativistic so that $|\hat{\mathbf{v}}| = 1$ and neutrino energy $E = |\mathbf{p}|$. The difficulty of solving Eq. (2) stems from the neutrino potential which couples neutrinos and antineutrinos of different momenta in the neutrino medium: [8–10]

$$\mathsf{H}_{\nu\nu} = \sqrt{2} G_\mathrm{F} \int \frac{\mathrm{d}^3 p'}{(2\pi)^3} (1 - \hat{\mathbf{v}} \cdot \hat{\mathbf{v}}') [\rho(t, \mathbf{x}, \mathbf{p}') - \bar{\rho}(t, \mathbf{x}, \mathbf{p}')], \tag{3}$$

where $G_\mathrm{F} \approx (293\ \mathrm{GeV})^{-2}$ is the Fermi coupling constant. When the neutrino potential is not negligible, neutrinos in a dense medium can oscillate in a collective manner (see, e.g., [11] for a review). In many cases collective oscillations cause neutrinos of different flavors to swap their spectra in certain energy ranges, a phenomenon dubbed as “stepwise spectral swap” or “spectral split” (e.g., [12–15]).

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Eq. (2) poses a difficult 7-dimensional problem with 1 temporal dimension, 3 spatial dimensions and 3 momentum dimensions. All existing work on collective neutrino oscillations has assumed certain directional symmetries in momentum space and/or spatial symmetries in position space to make this problem more tractable. For example, the spatial spherical symmetry and the directional axial symmetry (about the radial direction) are generally assumed for SN (e.g., [12–22]), and the spatial homogeneity and directional isotropy for the early universe (e.g., [23,24]). However, these spatial and directional symmetries are not necessarily preserved in collective neutrino oscillations. Imposing these symmetries may lead to results that are qualitatively different from those in real physical environments. It was recently shown that collective oscillations can break the directional axial symmetry in SN spontaneously [25,26], which obviously will not occur if this symmetry is artificially enforced. Similar result is also found in the neutrino media with an initial (approximate) isotropy [27]. A recent numeric study shows that the spatial homogeneity can be also broken in a toy model with 1 temporal and 1 spatial dimensions [28].

In this letter we propose to study the neutrino Line model with 2 spatial dimensions. The results derived from this simple model can provide valuable insights of the collective flavor transformation in the neutrino gas models of multiple spatial dimensions.

2. Neutrino Line model

We consider the time-independent (neutrino) Line model in which neutrinos are constantly emitted from the x axis or the (neutrino) Line and propagate inside the x–z plane. For simplicity, we assume that every point on the Line emits only neutrinos and antineutrinos of single energy E with intensities j_ν and $j_{\bar{\nu}}$, respectively, and in only two directions

$$\hat{\mathbf{v}}_\zeta = [u_\zeta, 0, v_z] \qquad (\zeta = L, R), \tag{4}$$

where $0 < v_z < 1$ and $u_R = -u_L = \sqrt{1-v_z^2}$. The Line model has 2 spatial dimensions (x, z) and 2 momentum dimensions (E, ζ) (because an antineutrino of energy E can be treated as a neutrino of energy $-E$ for the purpose of neutrino oscillations).

We will consider the scenario of two flavor mixing, e.g., between ν_e and ν_τ, in vacuum. In the mass basis

$$\mathsf{H}_0 = -\bar{\mathsf{H}}_0 = -\frac{\omega\eta}{2}\sigma_3, \tag{5}$$

where $\omega > 0$ is the vacuum neutrino oscillation frequency, $\eta = +1$ and -1 for NH and IH, respectively, and σ_3 is the third Pauli matrix.

We define reduced neutrino density matrices $\varrho \propto \rho$ and $\bar{\varrho} \propto \bar{\rho}$ which are normalized by condition

$$\mathrm{tr}\varrho = \mathrm{tr}\bar{\varrho} = 1. \tag{6}$$

The equations of motion for ϱ and $\bar{\varrho}$ in the Line model are

$$\mathrm{i}\hat{\mathbf{v}}_\zeta \cdot \nabla\varrho_\zeta(x,z) = [\mathsf{H}_0 + \mathsf{H}_{\nu\nu,\zeta}(x,z),\ \varrho_\zeta(x,z)], \tag{7a}$$

$$\mathrm{i}\hat{\mathbf{v}}_\zeta \cdot \nabla\bar{\varrho}_\zeta(x,z) = [-\mathsf{H}_0 + \mathsf{H}_{\nu\nu,\zeta}(x,z),\ \bar{\varrho}_\zeta(x,z)]. \tag{7b}$$

The neutrino potential in the above equation is

$$\mathsf{H}_{\nu\nu,\zeta}(x,z) = \mu[\varrho_{\tilde{\zeta}}(x,z) - \alpha\bar{\varrho}_{\tilde{\zeta}}(x,z)], \tag{8}$$

where

$$\mu = \sqrt{2}(1 - \hat{\mathbf{v}}_L \cdot \hat{\mathbf{v}}_R)G_F j_\nu \tag{9}$$

is the strength of the neutrino–neutrino coupling, $\tilde{\zeta} = R, L$ are the opposites of ζ, and $\alpha = j_{\bar{\nu}}/j_\nu$.

To facilitate numerical calculations we further impose a periodic condition on the x–z plane such that

$$\varrho_\zeta(x,z) = \varrho_\zeta(x+L,z), \qquad \bar{\varrho}_\zeta(x,z) = \bar{\varrho}_\zeta(x+L,z), \tag{10}$$

where L is the size of the periodic box. We define neutrino density matrices in the Fourier space as

$$\varrho_{\zeta,m}(z) = \frac{1}{L}\int_0^L e^{-\mathrm{i}mk_0x}\varrho_\zeta(x,z)\,\mathrm{d}x \tag{11}$$

such that

$$\varrho_\zeta(x,z) = \sum_m e^{\mathrm{i}mk_0x}\varrho_{\zeta,m}(z), \tag{12}$$

where m is an integer, and $k_0 = 2\pi/L$. We also define $\bar{\varrho}_{\zeta,m}(z)$ for the antineutrino in a similar way. Using Eq. (7) and

$$\hat{\mathbf{v}}_\zeta \cdot \nabla\varrho_\zeta = \sum_m e^{\mathrm{i}mk_0x}[v_z\varrho'_{\zeta,m} + \mathrm{i}mk_0u_\zeta\varrho_{\zeta,m}] \tag{13}$$

we obtain the equations of motion in the Fourier space:

$$\begin{aligned}\mathrm{i}v_z\varrho'_{\zeta,m} &= mk_0u_\zeta\varrho_{\zeta,m} + [\eta\omega\sigma_3/2,\ \varrho_{\zeta,m}] \\ &\quad + \mu\sum_{m'}[\varrho_{\tilde{\zeta},m'} - \alpha\bar{\varrho}_{\tilde{\zeta},m'},\ \varrho_{\zeta,m-m'}],\end{aligned} \tag{14a}$$

$$\begin{aligned}\mathrm{i}v_z\bar{\varrho}'_{\zeta,m} &= mk_0u_\zeta\bar{\varrho}_{\zeta,m} + [-\eta\omega\sigma_3/2,\ \bar{\varrho}_{\zeta,m}] \\ &\quad + \mu\sum_{m'}[\varrho_{\tilde{\zeta},m'} - \alpha\bar{\varrho}_{\tilde{\zeta},m'},\ \bar{\varrho}_{\zeta,m-m'}],\end{aligned} \tag{14b}$$

where $\varrho'_{\zeta,m} = \mathrm{d}\varrho_{\zeta,m}/\mathrm{d}z$.

3. Flavor instabilities

It is instructive to first review the flavor instability in the bipolar model with 1 spatial (or temporal) dimension and 1 momentum dimension [29–32]. This model can be obtained from the Line model by imposing the translation symmetry along the x axis and the left–right symmetry ($L \leftrightarrow R$) between the two angle directions. For the bipolar model Eq. (7) has solution

$$\varrho_\zeta(x,z) = e^{-\mathrm{i}\omega z\eta\sigma_3/2v_z}\varrho(0)e^{\mathrm{i}\omega z\eta\sigma_3/2v_z}, \tag{15a}$$

$$\bar{\varrho}_\zeta(x,z) = e^{-\mathrm{i}(-\omega)z\eta\sigma_3/2v_z}\bar{\varrho}(0)e^{\mathrm{i}(-\omega)z\eta\sigma_3/2v_z} \tag{15b}$$

in the absence of ambient neutrinos (i.e. $\mu = 0$), where $\varrho(0)$ and $\bar{\varrho}(0)$ are the neutrino density matrices at $z = 0$. In this vacuum oscillation solution an antineutrino behaves as a neutrino with a negative oscillation frequency $-\omega$ or negative energy $-E$. Inside the neutrino medium, however, ϱ and $\bar{\varrho}$ can oscillate with the same frequency Ω under suitable conditions such that

$$\varrho_\zeta(x,z) = e^{-\mathrm{i}\Omega z\eta\sigma_3/2v_z}\varrho(0)e^{\mathrm{i}\Omega z\eta\sigma_3/2v_z}, \tag{16a}$$

$$\bar{\varrho}_\zeta(x,z) = e^{-\mathrm{i}\Omega z\eta\sigma_3/2v_z}\bar{\varrho}(0)e^{\mathrm{i}\Omega z\eta\sigma_3/2v_z}, \tag{16b}$$

where Ω is a function of μ, α and ω [32]. This solution is equivalent to the precession motion of a pendulum in flavor space [31]. Similar solutions can exist for the scenarios with a continuous energy distribution of neutrinos [33]. When the neutrino mixing angle θ is small, the collective oscillation solution in Eq. (16) does not result in significant neutrino oscillations unless $\kappa = \mathrm{Im}(\Omega) > 0$. A positive κ indicates that the flavor pendulum cannot precess stably and must experience nutation in flavor space. This flavor instability can lead to collective neutrino oscillations with observable effects.

For the Line model the collective solution should take the form