Physics Letters B 747 (2015) 390–394

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

A new parameter in attractor single-field inflation

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ARTICLE INFO

Article history:
Received 24 February 2015
Received in revised form 11 June 2015
Accepted 11 June 2015
Available online 15 June 2015
Editor: S. Dodelson

ABSTRACT

We revisit the notion of slow-roll in the context of general single-field inflation. As a generalization of slow-roll dynamics, we consider an inflaton ϕ in an attractor phase where the time derivative of ϕ is determined by a function of ϕ, $\dot{\phi} = \dot{\phi}(\phi)$. In other words, we consider the case when the number of e-folds N counted backward in time from the end of inflation is solely a function of ϕ, $N = N(\phi)$. In this case, it is found that we need a new independent parameter to properly describe the dynamics of the inflaton field in general, in addition to the standard parameters conventionally denoted by ϵ, η, c_s^2 and s. Two illustrative examples are presented to discuss the non-slow-roll dynamics of the inflaton field consistent with observations.

1. Introduction

The primordial inflation [1] in the very early universe before the onset of the standard hot big bang evolution is now the leading candidate to explain otherwise extremely finely tuned initial conditions, such as the horizon and flatness problems. Furthermore, inflation can naturally provide a causal mechanism of producing primordial curvature perturbations that should have existed on super-horizon scales [2]. These primordial curvature perturbations are predicted to have a nearly scale invariant power spectrum and are statistically almost perfectly Gaussian. By recent observations including the Planck mission, these properties have been confirmed with very high accuracy [3–5].

While the inflationary picture itself is more and more supported and favored by recent observations, constructing a realistic and concrete model of inflation in the context of particle physics remains an open conundrum [6]. In this situation we should be open-minded and consider a wider, more general possibilities for inflation than the simplest model where a single, canonically normalized inflaton minimally coupled to Einstein gravity drives inflation. Such general theories may well predict verifiable new observational signatures such as a slight blue tilt for tensor perturbations [7] and suppression of the curvature perturbation on large scales [8]. We may have to take these possibilities more seriously, as the simplest possibilities including the $m^2\phi^2$ model seem to be not favored by the new Planck data [5].

A caution is in order when we study such general possibilities. We should keep in mind that many notions we have developed in the canonical models are not directly applicable to them. For example, the moment of horizon crossing which is crucial for standard single field inflation may not be as important as any other instants during inflation. This is because, contrary to the canonical model, the curvature perturbation may keep evolving on super-horizon scales until the end of inflation [9,10] by e.g. the existence of other relevant degrees of freedom, which may reflect the signatures of high energy physics [11]. In this article, we revisit the term “slow-roll” in the context of k-inflation type general $P(X,\phi)$ theory where $X \equiv -g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi/2$ [12].

The article is organized as follows. In Section 2 we extend the notion of slow-roll single-field inflation and consider the general, attractor phase inflation in the context of $P(X,\phi)$ theory. In describing the dynamics of the inflaton field, we introduce an independent new parameter p [see (10)] which identically vanishes in the canonical single-field model. The new parameter is slow-roll suppressed if the inflaton is slow-rolling. However, in the general case of attractor inflation where the inflaton is may not be slow-rolling, it may become of order unity. In Section 3 we present two examples to illustrate the possibility of the non-slow-roll dynamics consistent with the current observational constraints. We conclude the paper in Section 4.

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http://dx.doi.org/10.1016/j.physletb.2015.06.023

2. General attractor inflation

For $P(X,\phi)$ theory, the matter Lagrangian is given by

$$S_m = \int d^4x\sqrt{-g}P(X,\phi)\,. \tag{1}$$

This is the most general single scalar field action with their linear derivatives, which includes the standard canonical action $P = X - V$ and the Dirac–Born–Infeld type action. We assume that the inflaton is in an attractor phase, i.e., $\dot\phi$ is determined by a function of ϕ, but ϕ is not necessarily slowly evolving, as discussed in more detail below. Thus, in particular, we do not consider non-attractor inflation [14] where the dynamics depends both on ϕ and $\dot\phi$.

With the above Lagrangian, it is known that the spectral index of the curvature perturbation is given by [12]

$$n_{\mathcal{R}} - 1 = -2\epsilon - \eta - s\,, \tag{2}$$

as well as the running of the spectral index [13]

$$\alpha_{\mathcal{R}} = -2\epsilon\eta - \frac{\dot\eta}{H} - \frac{\dot s}{H}\,, \tag{3}$$

where

$$\begin{aligned} \epsilon &\equiv -\frac{\dot H}{H^2} = \frac{XP_X}{m_{\rm Pl}^2H^2}\,, \\ \eta &\equiv \frac{\dot\epsilon}{H\epsilon} = -\frac{\ddot H}{H^3\epsilon} + 2\epsilon\,, \\ s &\equiv \frac{\dot c_s}{Hc_s}\,, \end{aligned} \tag{4}$$

with the speed of sound c_s given by

$$c_s^{-2} = 1 + \frac{2XP_{XX}}{P_X}\,. \tag{5}$$

In deriving (2), it is assumed that H and c_s are slowly varying. The constrained value of $n_{\mathcal{R}} - 1 = 0.968 \pm 0.006$ [5] demands ϵ, η and s are all small, barring accidental cancellation among them. This situation is usually referred to as the “slow-roll” approximation. It is however quite misleading because the smallness of these parameters does not necessarily mean the inflaton is slowly evolving. This becomes more transparent if we consider the equation of motion for ϕ, which reads [12]

$$\frac{1}{a^3}\frac{d}{dt}\left(a^3P_X\dot\phi\right) = \frac{d}{dt}(P_X\dot\phi) + 3HP_X\dot\phi = P_\phi\,. \tag{6}$$

In the canonical case where $P_X = 1$, the smallness of ϵ and η would imply the smallness of the $\ddot\phi$ term in comparison with $3H\dot\phi$ term, which is the usual slow-roll approximation. But the second derivative term may not be negligible in the general $P(X,\phi)$ theory a priori.

Let us take another point of view by considering the second order component of the comoving curvature perturbation $\mathcal{R}$. In the context of the δN formalism [9,15] where $N = N(\phi)$, for single field case we can find (see for detail Appendix A)

$$\delta N = \mathcal{R} = \mathcal{R}_l\left[1 + \frac{1}{2}(\epsilon+\delta)\mathcal{R}_l + \cdots\right]\,, \tag{7}$$

where $\mathcal{R}_l \equiv -H\delta\phi/\dot\phi$ is the linear component of $\mathcal{R}$ with $\delta\phi$ being evaluated on flat slices at horizon crossing, and

$$\delta \equiv \frac{\ddot\phi}{H\dot\phi}\,. \tag{8}$$

The notion of “slow-roll”, i.e. slow evolution of the inflaton field is thus equivalent to requiring $|\delta| \ll 1$. Note that (7) is in fact the second order gauge transformation [16] and is independent of the structure of the matter sector, so should remain valid for $P(X,\phi)$ theory. Only when the kinetic sector is canonical we can use the relation $\dot H = -X/m_{\rm Pl}^2$ and find $\eta = 2(\epsilon+\delta)$, so that the smallness of the second order component of $\mathcal{R}$ in (7) is guaranteed. However, in $P(X,\phi)$ theory, $\dot H = -XP_X/m_{\rm Pl}^2$ so that in general we have

$$\eta = 2(\epsilon+\delta) + p\,, \tag{9}$$

where we have introduced a new parameter p defined by

$$p \equiv \frac{\dot P_X}{HP_X}\,. \tag{10}$$

Thus the coefficient in front of the second order component of $\mathcal{R}$ is not necessarily small.

Note that p may be expressed as

$$p = \delta\left(\frac{1}{c_s^2} - 1\right) + \frac{P_{X\phi}}{HP_X}\dot\phi = -3 - \delta + \frac{P_\phi}{H\dot\phi P_X}\,, \tag{11}$$

where for the second equality we have used the equation of motion (6). Equating these two expressions for p, we can eliminate HP_X and can write p as

$$p = \frac{\left(c_s^{-2}-1\right)\delta + 2(3+\delta)q}{1-2q} \quad \text{where} \quad q \equiv \frac{XP_{X\phi}}{P_\phi}\,. \tag{12}$$

This is another useful formula. Since p is expressed in terms of the cross derivative $P_{X\phi}$, we can see the qualitative dependence of p on how close the theory is to the canonical form where $P_{X\phi} = 0$. Explicitly, we can express p as

$$p \approx \begin{cases} \left(c_s^{-2}-1\right)\delta + \mathcal{O}(q) & \text{for } |q| \ll 1 \\ -3 - \delta + \mathcal{O}(q^{-1}) & \text{for } |q| \gg 1 \end{cases}\,. \tag{13}$$

Thus on general ground we expect that when $P(X,\phi)$ is highly non-canonical, we may have $|q| \gg 1$, and the slow-roll dynamics of the inflaton field is not guaranteed. In fact if $|q| \gg 1$, combined with (9), it is *required* that the non-slow-rollness must be as large as $\delta \approx 3$ with ϵ and η being kept small.

Before closing this section, let us reconsider the curvature perturbation expanded to second order (7) in the context of non-Gaussianity. Conventionally a local non-Gaussianity is represented by the non-linear parameter $f_{\rm NL}$ [17] which appears in the expansion as

$$\mathcal{R} = \mathcal{R}_l + \frac{3}{5}f_{\rm NL}\mathcal{R}_l^2 + \cdots\,. \tag{14}$$

For the canonical case, using (9), (7) reads

$$\mathcal{R} = \mathcal{R}_l + \frac{\eta}{4}\mathcal{R}_l^2 + \cdots\,, \tag{15}$$

which implies

$$f_{\rm NL} = \frac{5}{12}\eta\,. \tag{16}$$

This is in fact a half of the consistency relation for the squeezed limit of the bispectrum [18]. The remaining half $5\epsilon/6$ comes from the intrinsic non-Gaussianity of $\mathcal{R}_l$, which we have not taken into account here. See Appendix B for detail. However, given that for most inflationary models $\epsilon \ll 1$, (16) contributes more importantly to $f_{\rm NL}$.

Now, following the same step, from (9) we obtain for $P(X,\phi)$ theory,

$$f_{\rm NL} = \frac{5}{12}\left(\eta - \frac{p}{2}\right)\,. \tag{17}$$