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Criticality in the scale invariant standard model (squared)

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ABSTRACT

We consider first the standard model Lagrangian with μ_h^2 Higgs potential term set to zero. We point out that this classically scale invariant theory potentially exhibits radiative electroweak/scale symmetry breaking with very high vacuum expectation value (VEV) for the Higgs field, $\langle\phi\rangle \approx 10^{17-18}$ GeV. Furthermore, if such a vacuum were realized then cancellation of vacuum energy automatically implies that this nontrivial vacuum is degenerate with the trivial unbroken vacuum. Such a theory would therefore be critical with the Higgs self-coupling and its beta function nearly vanishing at the symmetry breaking minimum, $\lambda(\mu = \langle\phi\rangle) \approx \beta_\lambda(\mu = \langle\phi\rangle) \approx 0$. A phenomenologically viable model that predicts this criticality property arises if we consider two copies of the standard model Lagrangian, with exact Z_2 symmetry swapping each ordinary particle with a partner. The spontaneously broken vacuum can then arise where one sector gains the high scale VEV, while the other gains the electroweak scale VEV. The low scale VEV is perturbed away from zero due to a Higgs portal coupling, or via the usual small Higgs mass terms μ_h^2, which softly break the scale invariance. In either case, the cancellation of vacuum energy requires $M_t = (171.53 \pm 0.42)$ GeV, which is close to its measured value of (173.34 ± 0.76) GeV.

The discovery of a Higgs-like particle [1,2] with mass around 125 GeV confirms the standard picture of electroweak symmetry breaking via the nonzero vacuum expectation value of a scalar field [3]. This nontrivial vacuum arises from a Higgs potential, of the form:

$$V = \lambda\phi^\dagger\phi\phi^\dagger\phi - \mu_h^2\phi^\dagger\phi + h\mu_h^4 \,, \qquad (1)$$

where the $h\mu_h^4$ part is the cosmological constant (CC) term, usually neglected as it only affects gravitational physics, e.g. [4,5]. In models with classical scale invariance, however, the CC term is absent, as required by this symmetry. The physical cosmological constant still arises radiatively, but is a calculable function of the other parameters of the theory [6].

Interestingly, a possible hint of a deeper structure beyond the standard model has emerged in a rather unexpected manner. The Higgs quartic coupling when evolved up to a high scale $\sim 10^{17-18}$ GeV, appears to approximately satisfy: $\lambda = \dot{\lambda} (\equiv \beta_\lambda) = 0$ (for recent calculations, see [7,8]). This condition seems to be accidental, since it involves cancellation among numerically large quantities.[1] In this short note, we show that such a relation can naturally arise in scale invariant models as a consequence of setting the physical cosmological constant to its measured small value. This then automatically implies that two distinct vacua with broken and unbroken symmetries coexists in the theory, that is, the theory exhibits criticality. We consider cases of exact classical scale invariance and also the case where scale invariance is considered to be softly broken by the familiar μ_h^2 term in the Higgs potential.

Let us define the scale invariant standard model Lagrangian, $\mathcal{L}_{SM}^{SI}$, to be the same as the standard model Lagrangian except with the μ_h^2 term set to zero. The Higgs potential is then particularly simple:

$$V = \lambda\phi^\dagger\phi\phi^\dagger\phi \,. \qquad (2)$$

A Coleman–Weinberg analysis [12,13], reveals that such a potential, radiatively corrected, can exhibit spontaneous symmetry breaking. This requires $\lambda(\mu)$ to be small at some particular scale $\mu = \mu_1$, and μ_1 sets the scale of the VEV of ϕ. If we additionally require that the CC vanish, then we have much more stringent

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[1] Previous attempts to justify these conditions were based on somewhat obscure principle of multiple criticality [9] and still controversial proposal of asymptotic safety of gravity [10]. See also [11].

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constraints for spontaneous symmetry breaking to occur [6]. To explain what these are, let us first write the exact (all loop) effective potential as:

$$V = A(g_a(\mu), m_x(\mu), \mu)\phi^\dagger\phi\phi^\dagger\phi + B(g_a(\mu), m_x(\mu), \mu)\phi^\dagger\phi\phi^\dagger\phi \log\left(\frac{\phi^\dagger\phi}{\mu^2}\right) + C(g_a(\mu), m_x(\mu), \mu)\phi^\dagger\phi\phi^\dagger\phi \left[\log\left(\frac{\phi^\dagger\phi)}{\mu^2}\right)\right]^2 + \ldots , \quad (3)$$

where ... denotes all terms with higher-power logarithms and $g_a(\mu)$ and $m_x(\mu)$ denote all relevant running dimensionless couplings and effective masses. Then it can easily be shown that the requirement of a nontrivial vacuum, $\langle\phi\rangle \neq 0$, and vanishing CC require the condition:

$$A(g_a(\mu), m_x(\mu), \mu) = B(g_a(\mu), m_x(\mu), \mu) = 0 \quad (4)$$

to hold at the same renormalization scale $\mu = \mu_1$. Furthermore this renormalization scale defines the VEV of ϕ at that scale. Note that the renormalization scale independence of the effective potential implies that:

$$B(\mu = \mu_1) = \frac{1}{2}\mu \frac{dA}{d\mu}\bigg|_{\mu=\mu_1} , \qquad C(\mu = \mu_1) = \frac{1}{4}\mu \frac{dB}{d\mu}\bigg|_{\mu=\mu_1} . \quad (5)$$

The condition that $A = B = 0$ at the same renormalization scale $\mu = \mu_1$ implies that $\lambda \simeq 0$ and $\dot\lambda \simeq 0$ at that renormalization scale. Interestingly it has been observed that such a condition is nearly satisfied given the parameters of the standard model. However, one finds that the scale, μ_1 is very high, $\sim 10^{17-18}$ GeV, that is possibly as high as the Planck scale. Thus, it appears that the scale invariant standard model may exhibit spontaneous symmetry breaking, but with very high VEV scale, $\langle\phi\rangle \approx 10^{17-18}$ GeV.

This spontaneously broken phase, is degenerate with the unbroken phase where $\phi = 0$. Could it be possible that there is a second copy of the standard model, i.e. mirror model with exact Z_2 invariant Lagrangian [14], where the Z_2 symmetry is spontaneously broken? That is, where one copy gets a zero VEV and the other at a large scale, $\mu_1 \approx 10^{17-18}$ GeV? A small portal coupling $\kappa\phi^\dagger\phi\phi'^\dagger\phi'$ [the particles of the copy are denoted with a prime (′)] could then perturb the zero VEV of ϕ, to be nonzero, and thus be responsible for electroweak symmetry breaking. This could work if $\kappa > 0$ at the high scale, μ_1, and $\kappa < 0$ at the electroweak scale. Since the required value of κ is very small at the electroweak scale ($\kappa \sim 10^{-31\pm1}$), it is possible that quantum gravitational corrections contribute to the running of κ and could lead to its change in sign at the high scale cf. with the low scale.[2] Such a model could also be technically natural, despite the high scale (for discussions along these lines see [16,17] and references there-in).

The scale invariant standard model squared has Lagrangian:

$$\mathcal{L} = \mathcal{L}^{SI}_{SM}(e, u, d, \gamma, \ldots) + \mathcal{L}^{SI}_{SM}(e', u', d', \gamma', \ldots) + \mathcal{L}_{int} . \quad (6)$$

The Lagrangian $\mathcal{L}_{int}$ contains the Higgs portal interaction and, potentially, also gauge kinetic mixing. The potential for this scale invariant standard model squared is just the obvious generalization of Eq. (3), with A', B', C' parameterizing the corresponding terms for ϕ'. At the high scale, $\mu = \mu_1$, the quantum corrected effective potential has a particularly simple form. Given that the extremum and CC conditions require that $A' = B' = 0$ (at that scale), the Z_2 symmetry also implies $A = B = 0$, $C' = C$ (at that scale). It follows that

$$V = C\phi^\dagger\phi\phi^\dagger\phi \left[\log(\phi^\dagger\phi/\mu_1^2)\right]^2 + C\phi'^\dagger\phi'\phi'^\dagger\phi' \left[\log(\phi'^\dagger\phi'/\mu_1^2)\right]^2 + \kappa\phi'^\dagger\phi'\phi^\dagger\phi . \quad (7)$$

The leading order contribution to C arises at two loops and is given by:

$$C^{(2)}(\mu = \mu_1) = \frac{1}{64\pi^2\mu_1^4}\left[3\mathrm{Tr}m_V^4\gamma_V + \mathrm{Tr}m_S^4\gamma_S - 4\mathrm{Tr}m_F^4\gamma_F\right]\bigg|_{\mu=\mu_1} , \quad (8)$$

where $\gamma_x = \partial \ln m_x/\partial \ln\mu$ $(x = V, S, F)$. As discussed in the previous paragraph, we assume $\kappa > 0$ at the high scale $\mu = \mu_1$. If $C > 0$ at the scale $\mu = \mu_1$, the above potential has the spontaneously broken vacuum $\langle\phi'\rangle = \mu_1$, $\langle\phi\rangle = 0$ (where the VEV's are running parameters defined at the scale $\mu = \mu_1$), as well as a degenerate unbroken vacuum: $\langle\phi'\rangle = \langle\phi\rangle = 0$.

The Pseudo Goldstone Boson (PGB) of scale invariance, $m_{\mathrm{PGB}} = \frac{\partial^2 V}{\partial\phi_0'^2}\Big|_{\phi'=\mu_1}$ arises at the two-loop level,

$$m^2_{\mathrm{PGB}} = 4C(\mu = \mu_1)\mu_1^2 . \quad (9)$$

Considering only the dominant (W', Z' and t') contributions, we have

$$C^{(2)} = \frac{1}{64\pi^2\mu_1^4}\left\{6M_W^4\gamma_W + 3M_Z^4\gamma_Z - 12M_t^4\gamma_t\right\} = \frac{1}{64\pi^2\mu_1^4}\left\{6M_W^4\frac{\beta_{g_2}}{g_2} + 3M_Z^4\left[\cos^2\theta_w\frac{\beta_{g_2}}{g_2} + \sin^2\theta_w\frac{\beta_{g_1}}{g_2}\right] - 12M_t^4\frac{\beta_{y_t}}{y_t}\right\} \quad (10)$$

where g_1, g_2, y_t and the $U(1)$, $SU(2)$ gauge couplings and t Yukawa coupling ($\tan\theta_w \equiv g_1/g_2$) evaluated at the high scale, $\mu = \mu_1$. In deriving Eq. (10) we have used the relation $A(\mu_1) = B(\mu_1) = 0$. Also, the beta functions are defined by: $\beta_X \equiv \partial X/\partial\ln\mu$. Evaluating $C^{(2)}$ at the scale $\mu = \mu_1$ we find that $C^{(2)} \approx 2\times10^{-5}$, and thus the PGB mass is around $m_{\mathrm{PGB}} \approx 10^{-2}\mu_1 \sim 10^{16}$ GeV.

At the low scale $\mu \sim 100$ GeV, a non-zero VEV of ϕ is induced via the portal coupling κ, provided $\kappa < 0$ at this scale. Indeed at this low scale the part of the potential involving ϕ has the approximate form:

$$V = \lambda\phi^\dagger\phi\phi^\dagger\phi + \kappa\phi^\dagger\phi\phi'^\dagger\phi' . \quad (11)$$

We expect that $\langle\phi'\rangle$, evaluated as a function of renormalization scale, μ, does not greatly change in going from the high scale to the low scale. This means that the ϕ part of the potential, at the electroweak renormalization scale, is just:

$$V = \lambda\phi^\dagger\phi\phi^\dagger\phi - \mu_h^2\phi^\dagger\phi \quad (12)$$

where

$$\mu_h^2 = -\kappa\mu_1^2 = -\kappa(\langle\phi'\rangle)^2 . \quad (13)$$

[2] The one-loop gravitational correction to the beta-function for κ has the form: $c_1\kappa\mu^2/M_P^2 + c_2\mu^4/M_P^4$, where c_1 and c_2 are some constants and μ is the renormalization scale. Note that for sizeable κ the first term dominates and preserves $\kappa = 0$ fixed-point of non-gravitational renormalization group running (see, e.g., [15]). However, for tiny values of κ considered in this paper, the second term becomes significant even at scales well below the Planck mass, $\mu \gtrsim \kappa^{1/4}M_P$. If dominant, this contribution may indeed turn κ to be positive at high scales.