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A quantum reduction to spherical symmetry in loop quantum gravity

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ABSTRACT

Based on a recent purely geometric construction of observables for the spatial diffeomorphism constraint, we propose two distinct quantum reductions to spherical symmetry within full $3+1$-dimensional loop quantum gravity. The construction of observables corresponds to using the radial gauge for the spatial metric and allows to identify rotations around a central observer as unitary transformations in the quantum theory. Group averaging over these rotations yields our first proposal for spherical symmetry. Hamiltonians of the full theory with angle-independent lapse preserve this spherically symmetric subsector of the full Hilbert space. A second proposal consists in implementing the vanishing of a certain vector field in spherical symmetry as a constraint on the full Hilbert space, leading to a close analogue of diffeomorphisms invariant states. While this second set of spherically symmetric states does not allow for using the full Hamiltonian, it is naturally suited to implement the spherically symmetric midisuperspace Hamiltonian, as an operator in the full theory, on it. Due to the canonical structure of the reduced variables, the holonomy-flux algebra behaves effectively as a one parameter family of $2+1$-dimensional algebras along the radial coordinate, leading to a diagonal non-vanishing volume operator on 3-valent vertices. The quantum dynamics thus becomes tractable, including scenarios like spherically symmetric dust collapse.

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Introduction

Loop quantum gravity [1,2] as a whole has matured into a serious candidate theory for quantum gravity in recent years. Progress has been especially strong in the areas of computing black hole entropy [3–5] and studying quantisations of mini- [6] or midisuperspace [7] models using techniques from the full theory. On the other hand, it has been notoriously hard to extract physics from computations directly in the full $3+1$-dimensional theory. In order to avoid the "problem of time" associated with the underlying diffeomorphism invariance of general relativity, deparametrisation [8,9] has been introduced within loop quantum gravity [10–12] in order to obtain a true Hamiltonian evolution. A certain form of deparametrisation, however, always puts restrictions on the physical situation that one can describe, due to a, in general, finite range of physical coordinates. It is therefore desirable to have different deparametrisation techniques at one's disposal, tailored to different interesting physical problems. Furthermore, deparametrisation can in principle significantly alter the canonical structure, leading to different quantisation variables. While this does not happen for the standard example of dust [9], or only in a mild form of a possible rescaling for scalar fields [13] due to a Higgs-like "absorption" of matter degrees of freedom, we will encounter a more severe change of canonical structure due to a purely geometric deparametrisation in this article. As a direct consequence of this deparametrisation, we will obtain a family of holonomy-flux algebras labelled by the radial coordinate, each behaving effectively two-dimensional. Thus, we can use spin networks with three-valent vertices on which the volume operator is diagonal. The quantum dynamics thus becomes a lot more tractable than in the usual case. Also, the physical coordinate system introduced via this deparametrisation is ideally suited for introducing a quantum reduction to spherical symmetry. In order to simplify the presentation in this letter, we will gloss over some technical details which are addressed in our companion papers [14,15].

The radial gauge

Recently, a purely geometric construction of observables with respect to the spatial diffeomorphism constraint has been given [16], based on a physical coordinate system introduced by spatial geodesics outgoing from a central point σ_0. A point in the spatial slice Σ is uniquely defined via the exponential map

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$x^I \mapsto \exp_{\sigma_0}(x^I e^i_I)$, where x^I, $I = 1, 2, 3$ are coordinates in an internal space and e^i_I is a frame at σ_0 depending on a choice of a fiducial frame e^i_{0I} at σ_0 as well as the spatial metric q_{ij}. $i, j = 1, 2, 3$ are local tensor indices on Σ. It is convenient to switch to spherical coordinates $a, b = r, A, B$ in the internal space; $r = \sqrt{x^I x_I}$ being the radial coordinate, and A, B the angular coordinates, often abbreviated by θ. Once the tensor indices are adapted to the induced coordinate system on Σ $(i, j \to a, b)$, the observable Q_{ab} corresponding to the spatial metric satisfies $Q_{ra} = \delta_{ra}$ [17].

This construction of observables has an analogue in terms of a gauge fixing of the spatial diffeomorphism constraint, which we will employ in this paper. It is similar to the *radial gauge* employed in numerical relativity, see e.g. [18], which is why we adopt this terminology. We start with the ADM phase space, subject to the Poisson bracket $\{q_{ij}(\sigma), p^{kl}(\sigma')\} = \delta^k_{(i}\delta^l_{j)}\delta^{(3)}(\sigma, \sigma')$, and $\{\phi(\sigma), \pi(\sigma')\} = \delta^{(3)}(\sigma, \sigma')$ illustratively for generic matter fields, as well as the spatial diffeomorphism and Hamiltonian constraints $C_a = -2\nabla_b p^b{}_a + C^{\text{matter}}_a$ and H. We choose a reference metric $\check{q}_{ij}$, which in its own adapted coordinates $\check{a}, \check{b}$ automatically satisfies $\check{q}_{\check{r}\check{a}} = \delta_{\check{r}\check{a}}$. We now impose the constraint $q_{\check{r}\check{a}} = \delta_{\check{r}\check{a}}$, which enforces that the metrics q and $\check{q}$ differ at most in their (non-radial) A, B-components [16]. In order to show that $q_{\check{r}\check{a}} = \delta_{\check{r}\check{a}}$ is a good gauge fixing for C_a, we compute

$$\left\{q_{\check{r}\check{a}}(\sigma),\ C_b[M^b]\right\} = 2M_{(\check{r};\check{a})}(\sigma). \tag{1}$$

Indeed, the vector field M^b can always be chosen such that (1) is non-vanishing, since the equation $2M_{(\check{r};\check{a})} = \omega_{\check{r}\check{a}}$ is uniquely solvable for a given ω [16]. One can now pass to the Dirac bracket $\{\cdot,\cdot\}_{\text{DB}}$ implementing the constraints $C_b = 0$ and $q_{\check{r}\check{a}} = \delta_{\check{r}\check{a}}$. The details of this procedure are spelled out in our longer companion paper [14], since they are not essential for what follows. We find

$$\left\{q_{AB}(r,\theta), p^{CD}(r',\theta')\right\}_{\text{DB}} = \delta^C_{(A}\delta^D_{B)}\delta(r,r')\delta^{(2)}(\theta,\theta') \tag{2}$$

$$\left\{q_{\check{r}\check{a}}(r,\theta), *\right\}_{\text{DB}} = 0 \tag{3}$$

$$\left\{F(r,\theta), p^{\check{r}\check{a}}(r',\theta')\right\}_{\text{DB}} = -\mathcal{L}_{\underset{(r',\theta')}{\vec{M}}} F(r,\theta), \tag{4}$$

where $*$ denotes an arbitrary phase space function, F is any local function of $q_{AB}, p^{AB}, \phi, \pi$, and $\underset{(r',\theta')}{\vec{M}}$ is a vector field defined in [14], and $\mathcal{L}$ is a Lie derivative acting on the internal space of r, θ. We thus conclude that

1) the A, B components of the spatial metric and its momentum have canonical Dirac brackets,
2) the constraint $q_{\check{r}\check{a}} = \delta_{\check{r}\check{a}}$ is consistent with (3), and
3) $p^{\check{r}\check{a}}$ acts via infinitesimal spatial diffeomorphisms.

Passage to the reduced phase space, coordinatised by q_{AB} and p^{AB}, now requires us to solve the spatial diffeomorphism constraint for $p^{\check{r}\check{a}}$ and insert the resulting expression in the Hamiltonian. The details are provided in [14].

Quantisation

Starting from the Dirac brackets (2), we can construct SU(2) connection variables along standard lines, see e.g. [19] or our companion paper [15] for details. We first extend the phase space to the canonical pair $\left\{K^i_A(r,\theta), E^B_j(r',\theta')\right\} = \delta(r,r')\delta^{(2)}(\theta,\theta')\delta^B_A\delta^i_j$ subject to the additional Gauß constraint $G_{ij} := E^A_{[i}K_{A|j]} = 0$, where $i, j = 1, 2, 3$ are now SU(2) indices. Then, we perform a canonical transformation to the canonical pair $\left\{{}^{(\beta)}A^i_A(r,\theta), {}^{(\beta)}E^B_j(r',\theta')\right\} = \delta(r,r')\delta^{(2)}(\theta,\theta')\delta^B_A\delta^i_j$, where ${}^{(\beta)}A^i_A = \frac{1}{2}\epsilon^{ijk}\Gamma_{Ajk} + \beta K^i_A$, Γ_{Ajk} is the Peldan hybrid spin connection [20], and β is a free parameter, similar to the Barbero–Immirzi parameter, and ${}^{(\beta)}E^A_i := E^A_i/\beta$. From these variables, we construct holonomies and fluxes as

$$h_e(A) := \mathcal{P}\exp\left(-\int_e {}^{(\beta)}A_{Ai}\tau^i dx^A\right) \tag{5}$$

$$E_n(S) := \int_S {}^{(\beta)}E^A_i(\sigma)n^i(\sigma)\epsilon_{AB}\, dr \wedge dx^B, \tag{6}$$

where the n^i are Lie algebra valued smearing functions and τ^i the Pauli matrices. We emphasise that the paths e are tangential to the spheres $S^2_r \subset \Sigma$ of constant r, while the surfaces S are foliated by radial geodesics. The reduced phase space, labelled by the variables q_{AB} and p^{CD}, has thus been reexpressed via holonomies and fluxes with restricted path and surface labels. Up to this restriction, the corresponding holonomy-flux algebra is identical to the one from standard loop quantum gravity. Thus, quantisation can proceed along standard lines, see e.g. [2], resulting in an L^2 space over the space of *restricted* generalised connections $\bar{\mathcal{A}}_{\text{res}}$, meaning with restricted paths as above. A generic element in this Hilbert space, a cylindrical function, will then depend on a finite number of holonomies, and consequently have support only at a finite number of radial distances r from σ_0. A basis in the space of gauge invariant cylindrical functions is thus given by cylindrical functions depending on spin networks embedded in the spheres S^2_r. We will call such a basis element *multi spin network*.

Geometric operators

We will focus the discussion about quantum operators in this paper on the volume operator, since its construction highlights the main peculiarities coming from the radial gauge. The volume operator [21] plays a pivotal role in the definition of the LQG dynamics [22], where it enters through Poisson bracket identities, known as "Thiemann's tricks". We will employ a similar construction. Details of the construction of the Hamiltonian are provided in [15].

Since the delta-distribution in (2) is 3-dimensional, we also need to define a 3-volume operator in order to use an identity such as $e^i_A(\sigma) = \left\{A^i_A(\sigma), V_\Delta\right\}$, where V_Δ is the volume of a small open region containing σ. Classically, the volume of a region R is given by $V(R) = \int_R \sqrt{q}\, d^3x$ with

$$\sqrt{q} = \sqrt{V^i V_i}, \qquad V^i := \frac{\beta^2}{2}\epsilon^{ijk\,(\beta)}E^{A\,(\beta)}_j E^B_k \epsilon_{AB}. \tag{7}$$

We now try to promote this expression to an operator along the lines of [23,24], that is we choose a lattice approximation of R, approximate the integrant by fluxes, and compute the action of the resulting operator in the limit of an infinitely refined lattice. A subtlety occurs when trying to remove the regulators of a given discretisation: while the integral provides three powers of the lattice spacing, two in the tangential directions and one in the radial direction, the fluxes under the integral absorb four powers, two in the tangential direction, and two in the radial direction. This means that we are left with an unused power of the radial lattice spacing. If this were a coordinate distance, the resulting operator would have the unacceptable property of being coordinate dependent. However, the radial distance is a physical distance, due to our chosen deparametrisation. The volume operator thus has a residual dependence on a *physical* regulator. We propose the following natural choice of a radial lattice to deal with the above problem: we choose a set of real positive numbers l_i, $i \in \mathbb{N}_0$ with $l_0 = 0$, which define the extent of the radial smearing of fluxes (the dual lattice), and set $\Delta l_i := l_i - l_{i-1}$. Next, we choose real positive numbers r_i,

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